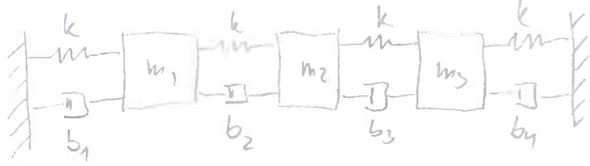
MAMS – cvičenie 4

Pr 1



$m_1 = 2\,kg$ | $k = 10^5\ N/m$

$m_2 = 5\,kg$ | $\mu = 0{,}1$

$m_3 = 1\,kg$ | $\gamma = 0{,}0001$

$$m_1 \ddot{x}_1 = -k x_1 + k(x_2 - x_1)$$

$$m_2 \ddot{x}_2 = -k(x_2 - x_1) + k(x_3 - x_2)$$

$$m_3 \ddot{x}_3 = -k(x_3 - x_2) - k x_3$$

$$m_1 \ddot{x}_1 + 2k x_1 - k x_2 = 0$$

$$m_2 \ddot{x}_2 - k x_1 + 2k x_2 - k x_3 = 0$$

$$m_3 \ddot{x}_3 - k x_2 + 2k x_3 = 0$$

$$[M] = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} ; \quad [K] = \begin{bmatrix} 2k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & 2k \end{bmatrix}$$

$$[B] = \mu[M] + \gamma[K]$$

– homogénne rov. sústavy bez tlmenia: $\left([K] - \underbrace{\omega^2}_{\lambda}[M]\right)\{X\} = \{0\}$ $\left(\text{matlab } \left([M]^{-1}[K] - \lambda[I]\right)\right)$

$$\Downarrow$$

$$[\lambda] = [\Omega_0^2] \;;\; [\Psi]$$

– ortogonalita → matice modál. hmot. a tuhosti

$$[\tilde{M}] = [\Psi]^T[M][\Psi]$$

$$[\tilde{K}] = [\Psi]^T[K][\Psi]$$

$$[\tilde{B}] = [\Psi]^T[B][\Psi]$$

– normované vl. tvary: $[\Phi] = [\Psi][\tilde{M}^{-\frac{1}{2}}]$

– modálna transformácia: $[\tilde{M}]\{\ddot{q}\} + [\tilde{B}]\{\dot{q}\} + [\tilde{K}]\{q\} = \{0\}$

$$\tilde{M}_r \ddot{q}_r + \tilde{B}_r \dot{q}_r + \tilde{K}_r q_r = 0 \;. \quad ; r = 1, 2, 3$$

$$\tilde{M}_r s^2 + \tilde{B}_r s + \tilde{K}_r = 0 \;\Rightarrow\; s_{1,2r} = \frac{-\tilde{B}_r \pm \sqrt{\tilde{B}_r^2 - 4\tilde{M}_r \tilde{K}_r}}{2\tilde{M}_r} \quad \begin{matrix} \nearrow s_{1r} = -\delta_r + i\Omega_{dr} \\ \searrow s_{2r} = -\delta_r - i\Omega_{dr} \end{matrix}$$

$$\delta_r = \frac{\tilde{B}_r}{\tilde{M}_r}$$

$$\Omega_{0r}^2 = \frac{\tilde{K}_r}{\tilde{M}_r}$$

$$\Omega_{dr} = \Omega_{0r}\sqrt{1 - \xi_r^2}$$

$$\xi_r = \frac{\delta_r}{\Omega_{0r}} = \frac{\mu}{2\Omega_{0r}} + \frac{1}{2}\gamma\,\Omega_{0r}$$

má korene $S_{1,2}$ v matlabe :

$$\{\ddot{q}\} + [\tilde{M}]^{-1}[\tilde{B}]\{\dot{q}\} + [\tilde{M}]^{-1}[\tilde{K}]\{q\} = \{0\}$$

$$\{\ddot{q}\} = -[\tilde{M}]^{-1}[\tilde{B}]\{\dot{q}\} - [\tilde{M}]^{-1}[\tilde{K}]\{q\}$$

$$\{\dot{q}\} = [I]\{\dot{q}\} + [0]\{q\}$$

$$\begin{bmatrix} \{\ddot{q}\} \\ \{\dot{q}\} \end{bmatrix} = \underbrace{\begin{bmatrix} -[\tilde{M}]^{-1}[\tilde{B}] & -[\tilde{M}]^{-1}[\tilde{K}] \\ [I] & [0] \end{bmatrix}}_{[A]} \begin{bmatrix} \{\dot{q}\} \\ \{q\} \end{bmatrix}$$

$[A] \rightarrow$ rozložíme na vl. čísla

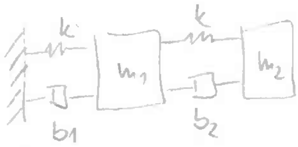
– matica receptancie:

$$[\alpha(\omega)] = [\Phi]\Big(\underbrace{[\Omega_0^2]}_{[\lambda]} + 2i\omega \underbrace{\mathrm{Re}[\lambda]}_{[\delta]} - \omega^2[I]\Big)^{-1}[\Phi]^T$$

– mat. pohyblivosti: $[Y(\omega)] = i\omega[\alpha(\omega)]$

– mat. inertancie: $[A(\omega)] = i\omega[Y(\omega)] = -\omega^2[\alpha(\omega)]$

Pr. 2



$m_1 = 5\,kg$

$m_2 = 2\,kg$

$k = 10^5\ N/m$

$b_1 = 10\,kg/s$

$b_2 = 20\,kg/s$

$$[M] = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$$

$$[K] = \begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix}$$

$$[B] = \begin{bmatrix} b_1+b_2 & -b_2 \\ -b_2 & b_2 \end{bmatrix}$$

$$[\bar{M}] = \begin{bmatrix} [0] & [M] \\ [M] & [B] \end{bmatrix} \; ; \; [\bar{K}] = \begin{bmatrix} -[M] & [0] \\ [0] & [K] \end{bmatrix}$$

$$[A] = -[\bar{M}]^{-1}[\bar{K}] \rightarrow \text{vl. čísla} \begin{Bmatrix} \lambda_r \\ \lambda_r^* \end{Bmatrix} = -\delta_r \pm i\Omega_{dr} \rightarrow \begin{matrix} \{\bar{X}\}_r \\ \{\bar{X}^*\}_r \end{matrix} \; ; \; \begin{matrix} \{\underline{X}\}_r \\ \{\underline{X}^*\}_r \end{matrix}$$

– ortogonalita
$$[\bar{X}]^T[\bar{M}][\bar{X}] = [\tilde{M}]$$
$$[\bar{X}]^T[\bar{K}][\bar{X}] = [\tilde{K}]$$

– normovanie:
$$[\bar{\phi}] = [\bar{X}][\tilde{M}]^{-\frac{1}{2}}$$
$$[\underline{\phi}] = [\underline{X}][\tilde{m}^{-\frac{1}{2}}]$$

– mat. receptancie:
$$[\alpha(\omega)] = [\phi][D][\underline{\phi}]^T + [\phi^*][D^*][\underline{\phi}^*]^T$$

PR3 Určte vlastné frekvencie a tvary kmitov nosníka metódou konečných prvkov pre rôzne okrajové podm.

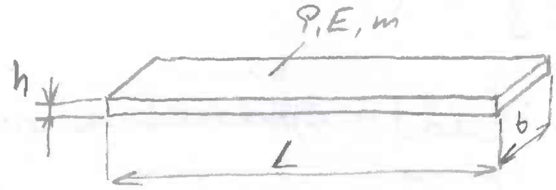
D: $L = 1\,m$

$b = 0{,}03\,m$

$h = 0{,}01\,m$

$E = 205 \cdot 10^9\,Pa$

$\rho = 7850\,kg \cdot m^{-3}$



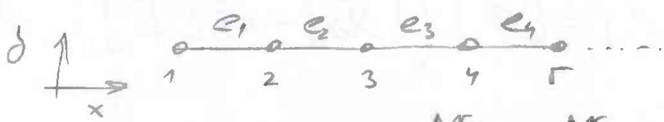
$m = \rho . V = \rho . bhL = \rho . A L \;;\; A = b . h$

$I = \dfrac{bh^3}{12}$

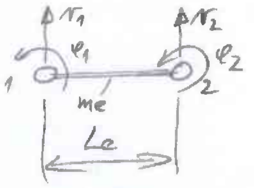
1.) voľný nosník

2.) jednostranne votknutý

3.) obojstranne —"—



jeden BEAM prvok:



→ 4° voľnosti

umožňuje prenášať iba ohybové zaťaženie v rovine

- matica tuhosti 1 elementu:

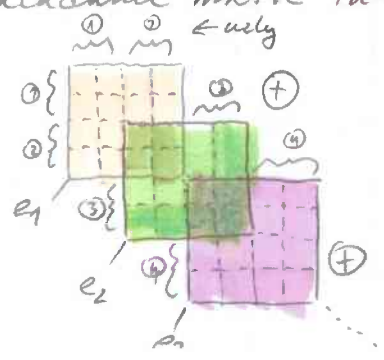
$$[k]_e = \frac{E . I}{L_e^3} \cdot \begin{bmatrix} 12 & 6L_e & -12 & 6L_e \\ 6L_e & 4L_e^2 & -6L_e & 2L_e^2 \\ -12 & -6L_e & 12 & -6L_e \\ 6L_e & 2L_e^2 & -6L_e & 4L_e^2 \end{bmatrix} \;; \text{ kde } L_e = \frac{L}{n_e}$$

n_e - počet elementov

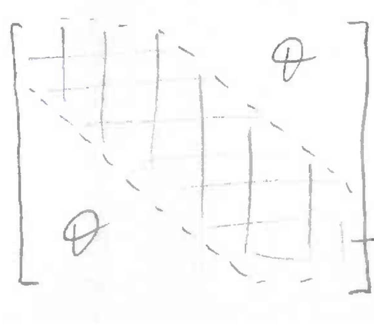
- matica hmotnosti 1 elementu:

$$[m]_e = \frac{m_e}{420} \cdot \begin{bmatrix} 156 & 22L_e & 54 & -13L_e \\ 22L_e & 4L_e^2 & 13L_e & -3L_e^2 \\ 54 & 13L_e & 156 & -22L_e \\ -13L_e & -3L_e^2 & -22L_e & 4L_e^2 \end{bmatrix} \;; \text{ kde } m_e = \frac{m}{n_e}$$

- skladanie matíc tuhosti/hmotnosti ⇒ globálne matice



→



→ pásová štruktúra

- Riešenie metódou rozkladu matice na vlastné čísla a vlastné vektory:

$$[A] = [M]^{-1} \cdot [K]$$

$$eig([A]) \rightarrow [\Omega_0^2] \text{ - spektrálna matica}$$

$$[\Psi] \text{ - modálna matica}$$

- Matica funkcií frekvenčných odoziev

$$[\alpha] = [\phi]\left([\Omega_0^2] - \omega^2 [I]\right)^{-1} [\phi]^T$$