$$\begin{pmatrix} A_x \\ A_y \\ A_z \\ T \\ M_y \\ M_z \end{pmatrix} = \text{Find}(A_x, A_y, A_z, T, M_y, M_z) \qquad \begin{pmatrix} A_x \\ A_y \\ A_z \\ T \end{pmatrix} = \begin{pmatrix} 0.0 \\ -93.2 \\ 57.1 \\ 47.1 \end{pmatrix} \text{lb} \qquad \begin{pmatrix} M_y \\ M_z \end{pmatrix} = \begin{pmatrix} 0.00 \\ 0.00 \end{pmatrix} \text{lb}\cdot\text{ft}$$

Problem 21-61

Show that the angular velocity of a body, in terms of Euler angles ϕ, θ and ψ may be expressed as $\boldsymbol{\omega} = (\phi' \sin\theta \sin\psi + \theta' \cos\psi)\,\mathbf{i} + (\phi' \sin\theta \cos\psi - \theta' \sin\psi)\,\mathbf{j} + (\phi' \cos\theta + \psi')\,\mathbf{k}$, where **i**, **j**, and **k** are directed along the *x*, *y*, *z* axes as shown in Fig. 21-15*d*.

Solution:

From Fig. 21 - 15*b*, due to rotation ϕ, the *x*, *y*, *z* components of ϕ' are simply ϕ' along *z* axis

From Fig. 21 - 15*c*, due to rotation θ, the *x*, *y*, *z* components of ϕ' and θ' are $\phi' \sin\theta$ in the *y* direction, $\phi' \cos\theta$ in the *z* direction, and θ' in the *x* direction.

Lastly, rotation ψ, Fig 21 - 15*d*, produces the final components which yields

$$\boldsymbol{\omega} = (\phi' \sin(\theta) \sin(\psi) + \theta' \cos(\psi))\mathbf{i} + (\phi' \sin(\theta) \cos(\psi) - \theta' \sin(\psi))\mathbf{j} + (\phi' \cos(\theta) + \psi')\mathbf{k}$$

Q.E.D

Problem 21-62

A thin rod is initially coincident with the *Z* axis when it is given three rotations defined by the Euler angles ϕ, θ, and ψ. If these rotations are given in the order stated, determine the coordinate direction angles α, β, γ of the axis of the rod with respect to the *X*, *Y*, and *Z* axes. Are these directions the same for any order of the rotations? Why?

Given:

$\phi = 30$ deg

$\theta = 45$ deg

$\psi = 60$ deg

Solution:

$$\mathbf{u} = \begin{pmatrix} \cos(\phi) & -\sin(\phi) & 0 \\ \sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \text{acos}(\mathbf{u}) \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 69.3 \\ 127.8 \\ 45.0 \end{pmatrix} \text{deg}$$

The last rotation (ψ) does not affect the result because the rod just spins around its own axis.

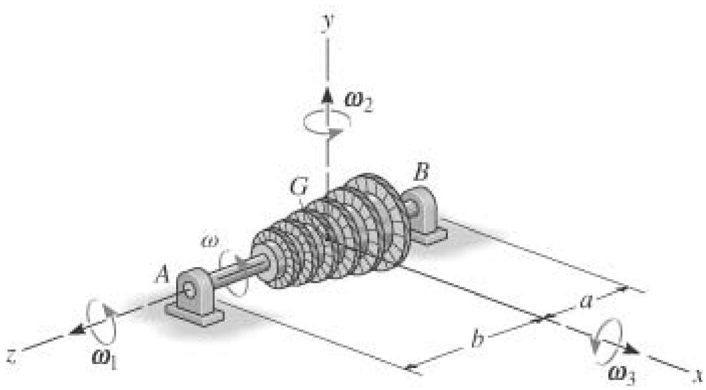
The order of application of the rotations does affect the final result since rotational position is not a vector quantity.

Problem 21-63

The turbine on a ship has mass M and is mounted on bearings A and B as shown. Its center of mass is at G, its radius of gyration is k_z, and $k_x = k_y$. If it is spinning at angular velocity ω, determine the vertical reactions at the bearings when the ship undergoes each of the following motions: (a) rolling ω_1, (b) turning ω_2, (c) pitching ω_3.

Units Used:

$\text{kN} = 1000 \text{ N}$



Given:

$M = 400 \text{ kg}$ $\qquad k_x = 0.5 \text{ m}$

$\omega = 200 \dfrac{\text{rad}}{\text{s}}$ $\qquad k_z = 0.3 \text{ m}$

$\omega_1 = 0.2 \dfrac{\text{rad}}{\text{s}}$ $\qquad a = 0.8 \text{ m}$

$\omega_2 = 0.8 \dfrac{\text{rad}}{\text{s}}$ $\qquad b = 1.3 \text{ m}$

$\omega_3 = 1.4 \dfrac{\text{rad}}{\text{s}}$

Solution:

$$\mathbf{I_G} = M\begin{pmatrix} k_x^2 & 0 & 0 \\ 0 & k_x^2 & 0 \\ 0 & 0 & k_z^2 \end{pmatrix}$$

Guesses $\quad A_x = 1 \text{ N} \quad A_y = 1 \text{ N}$

$B_x = 1 \text{ N} \quad B_y = 1 \text{ N}$

(a) Rolling Given

$$\begin{pmatrix} A_x \\ A_y \\ 0 \end{pmatrix} + \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -M g \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \text{N}$$

$$\begin{pmatrix} 0 \\ 0 \\ b \end{pmatrix} \times \begin{pmatrix} A_x \\ A_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -a \end{pmatrix} \times \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} = \mathbf{I_G} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \frac{\text{rad}}{\text{s}^2} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\mathbf{I_G} \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \right]$$

$$\begin{pmatrix} A_x \\ A_y \\ B_x \\ B_y \end{pmatrix} = \text{Find}(A_x, A_y, B_x, B_y) \qquad \begin{pmatrix} A_x \\ B_x \end{pmatrix} = \begin{pmatrix} 0.00 \\ 0.00 \end{pmatrix} \text{kN} \qquad \begin{pmatrix} A_y \\ B_y \end{pmatrix} = \begin{pmatrix} 1.50 \\ 2.43 \end{pmatrix} \text{kN}$$

(b) Turning Given

$$\begin{pmatrix} A_x \\ A_y \\ 0 \end{pmatrix} + \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -M g \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \text{N}$$

$$\begin{pmatrix} 0 \\ 0 \\ b \end{pmatrix} \times \begin{pmatrix} A_x \\ A_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -a \end{pmatrix} \times \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} = \mathbf{I_G} \begin{pmatrix} \omega \omega_2 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \omega_2 \\ \omega \end{pmatrix} \times \left[\mathbf{I_G} \begin{pmatrix} 0 \\ \omega_2 \\ \omega \end{pmatrix} \right]$$

$$\begin{pmatrix} A_x \\ A_y \\ B_x \\ B_y \end{pmatrix} = \text{Find}(A_x, A_y, B_x, B_y) \qquad \begin{pmatrix} A_x \\ B_x \end{pmatrix} = \begin{pmatrix} 0.00 \\ 0.00 \end{pmatrix} \text{kN} \qquad \begin{pmatrix} A_y \\ B_y \end{pmatrix} = \begin{pmatrix} -1.25 \\ 5.17 \end{pmatrix} \text{kN}$$

(c) Pitching Given

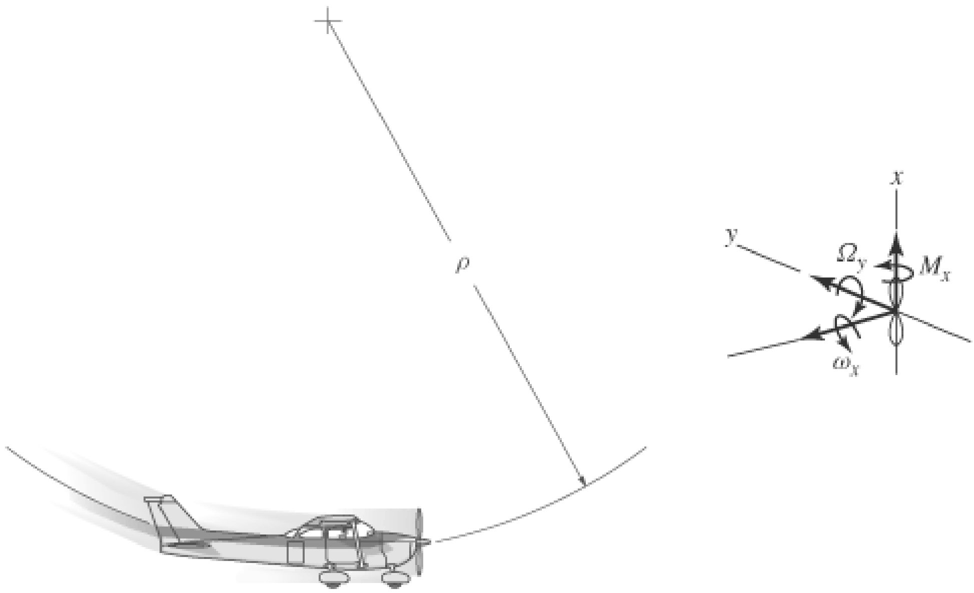
$$\begin{pmatrix} A_x \\ A_y \\ 0 \end{pmatrix} + \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -M g \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \text{N}$$

$$\begin{pmatrix} 0 \\ 0 \\ b \end{pmatrix} \times \begin{pmatrix} A_x \\ A_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -a \end{pmatrix} \times \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} = \mathbf{I_G} \begin{pmatrix} 0 \\ -\omega\,\omega_3 \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_3 \\ 0 \\ \omega \end{pmatrix} \times \left[\mathbf{I_G} \begin{pmatrix} \omega_3 \\ 0 \\ \omega \end{pmatrix} \right]$$

$$\begin{pmatrix} A_x \\ A_y \\ B_x \\ B_y \end{pmatrix} = \text{Find}(A_x, A_y, B_x, B_y) \qquad \begin{pmatrix} A_x \\ B_x \end{pmatrix} = \begin{pmatrix} -4.80 \\ 4.80 \end{pmatrix} \text{kN} \qquad \begin{pmatrix} A_y \\ B_y \end{pmatrix} = \begin{pmatrix} 1.50 \\ 2.43 \end{pmatrix} \text{kN}$$

***Problem 21-64**

An airplane descends at a steep angle and then levels off horizontally to land. If the propeller is turning clockwise when observed from the rear of the plane, determine the direction in which the plane tends to turn as caused by the gyroscopic effect as it levels off.

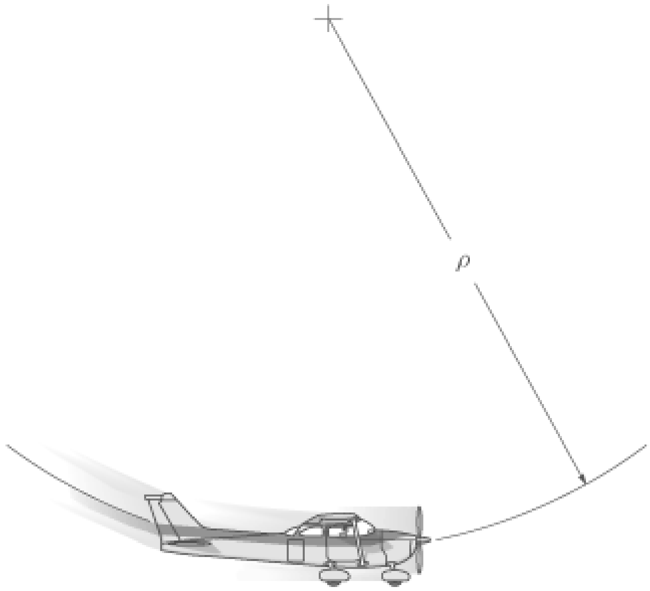


Solution:

As noted on the diagram M_x represents the effect of the plane on the propeller. The opposite effect occurs on the plane. Hence, the plane tends to **turn to the right when viewed from above.**

Problem 21-65

The propeller on a single-engine airplane has a mass M and a centroidal radius of gyration k_G computed about the axis of spin. When viewed from the front of the airplane, the propeller is turning clockwise at ω_s about the spin axis. If the airplane enters a vertical curve having a radius ρ and is traveling at speed v, determine the gyroscopic bending moment which the propeller exerts on the bearings of the engine when the airplane is in its lowest position.



Given:

$M = 15\ \text{kg}$

$k_G = 0.3\ \text{m}$

$v = 200\ \frac{\text{km}}{\text{hr}}$

$\omega_s = 350\ \frac{\text{rad}}{\text{s}}$

$\rho = 80\ \text{m}$

Solution:

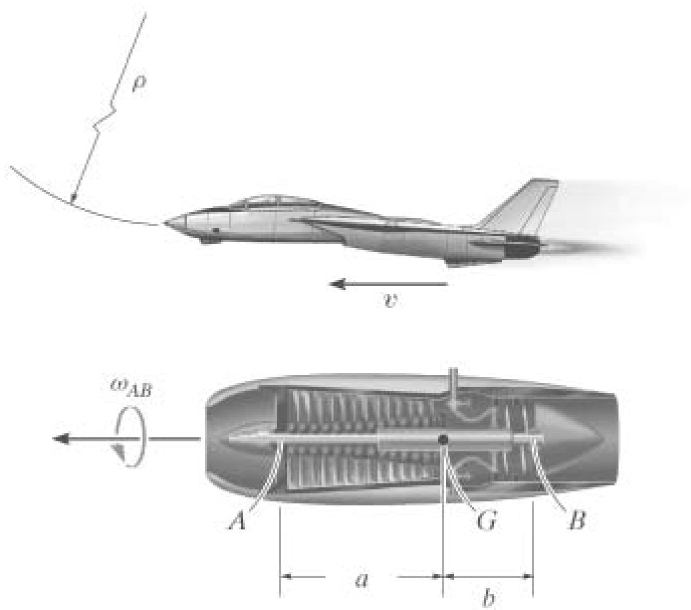
$\Omega_y = \frac{v}{\rho}$

$M_z = \left(M k_G^{\,2}\right)\Omega_y \omega_s$

$M_z = 328\ \text{N}\cdot\text{m}$

Problem 21-66

The rotor assembly on the engine of a jet airplane consists of the turbine, drive shaft, and compressor. The total mass is m_r, the radius of gyration about the shaft axis is k_{AB}, and the mass center is at G. If the rotor has an angular velocity ω_{AB}, and the plane is pulling out of a vertical curve while traveling at speed v, determine the components of reaction at the bearings A and B due to the gyroscopic effect.



Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

$m_r = 700\ \text{kg}$

$k_{AB} = 0.35\ \text{m}$

$\omega_{AB} = 1000\ \frac{\text{rad}}{\text{s}}$

$\rho = 1.30 \text{ km}$

$a = 0.8 \text{ m}$

$b = 0.4 \text{ m}$

$v = 250 \dfrac{\text{m}}{\text{s}}$

Solution: $M = m_r k_{AB}{}^2 \omega_{AB} \dfrac{v}{\rho}$

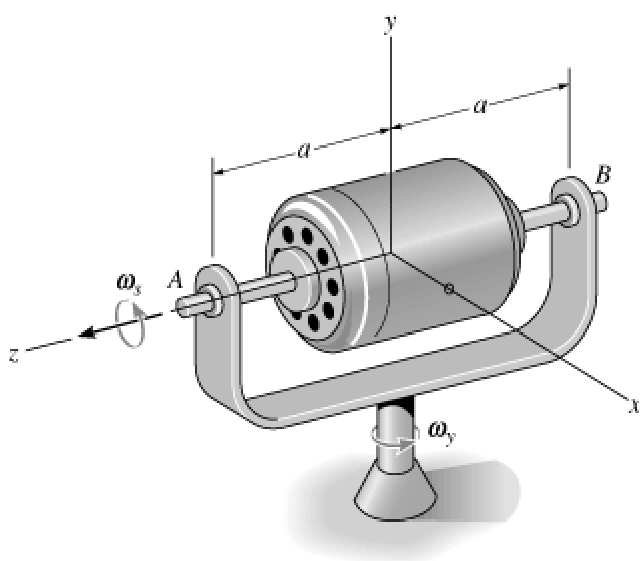
Guesses $A = 1 \text{ N}$ $B = 1 \text{ N}$

Given $A a - B b = M$ $A + B = 0$ $\begin{pmatrix} A \\ B \end{pmatrix} = \text{Find}(A, B)$ $\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 13.7 \\ -13.7 \end{pmatrix} \text{kN}$

Problem 21-67

A motor has weight W and has radius of gyration k_z about the z axis. The shaft of the motor is supported by bearings at A and B, and is turning at a constant rate $\omega_s = \omega_z\mathbf{k}$, while the frame has an angular velocity of $\omega_y = \omega_y\mathbf{j}$. Determine the moment which the bearing forces at A and B exert on the shaft due to this motion.

Given:



$W = 50 \text{ lb}$

$k_z = 0.2 \text{ ft}$

$\omega_z = 100 \dfrac{\text{rad}}{\text{s}}$

$\omega_y = 2 \dfrac{\text{rad}}{\text{s}}$

$a = 0.5 \text{ ft}$

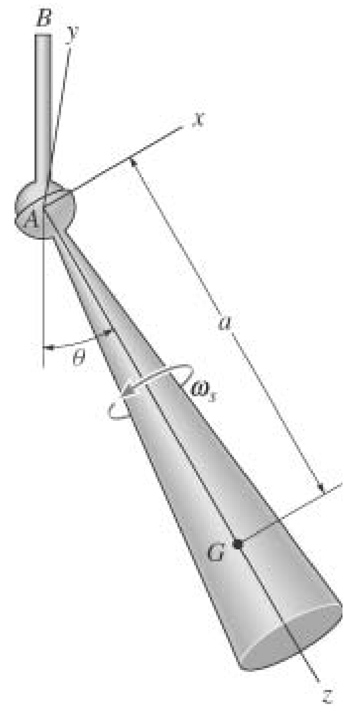
$g = 32.2 \dfrac{\text{ft}}{\text{s}^2}$

Solution:

$$\mathbf{M} = \begin{pmatrix} 0 \\ \omega_y \\ 0 \end{pmatrix} \times \begin{bmatrix} 0 \\ 0 \\ \left(\dfrac{W}{g}\right) k_z{}^2 \omega_z \end{bmatrix} \qquad \mathbf{M} = \begin{pmatrix} 12.4 \\ 0.0 \\ 0.0 \end{pmatrix} \text{lb} \cdot \text{ft}$$

***Problem 21-68**

The conical top has mass M, and the moments of inertia are $I_x = I_y$ and I_z. If it spins freely in the ball-and-socket joint at A with angular velocity ω_s compute the precession of the top about the axis of the shaft AB.



Given:

$M = 0.8\ \text{kg}$ $\quad a = 100\ \text{mm}$

$I_x = 3.5\ 10^{-3}\ \text{kg}\cdot\text{m}^2$ $\quad \theta = 30\ \text{deg}$

$I_z = 0.8 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ $\quad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

$\omega_s = 750\ \dfrac{\text{rad}}{\text{s}}$

Solution: Using Eq. 21-30.

$$\Sigma M_x = -I_x \phi'^2 \sin(\theta)\cos(\theta) + I_z\phi' \sin(\theta)(\phi' \cos(\theta) + \psi')$$

Guess $\phi' = 1\ \dfrac{\text{rad}}{\text{s}}$

Given $M g \sin(\theta) a = -I_x \phi'^2 \sin(\theta)\cos(\theta) + I_z\phi' \sin(\theta)(\phi' \cos(\theta) + \omega_s)$

$\phi' = \text{Find}(\phi')$ $\quad \phi' = 1.31\ \dfrac{\text{rad}}{\text{s}}$ $\quad$ low precession

Guess $\phi' = 200\ \dfrac{\text{rad}}{\text{s}}$

Given $M g \sin(\theta) a = -I_x \phi'^2 \sin(\theta)\cos(\theta) + I_z\phi' \sin(\theta)(\phi' \cos(\theta) + \omega_s)$

$\phi' = \text{Find}(\phi')$ $\quad \phi' = 255\ \dfrac{\text{rad}}{\text{s}}$ $\quad$ high precession

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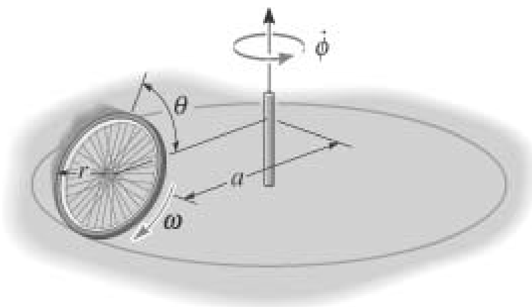
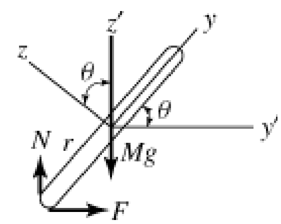
Problem 21-69

A wheel of mass m and radius r rolls with constant spin ω about a circular path having a radius a. If the angle of inclination is θ, determine the rate of precession. Treat the wheel as a thin ring. No slipping occurs.





Solution:

Since no sipping occurs, $r\psi' = a + r\cos(\theta)\phi'$ $\psi' = \left(\frac{a + r cos(\theta)}{r}\right)\phi'$

Also, $\omega = \phi' + \psi'$ $F = m\left(a\phi'^2\right)$ $N - mg = 0$

$$I_x = I_y = \frac{mr^2}{2} \qquad I_z = mr^2$$

$$\omega = \phi'\sin(\theta)\mathbf{j} + (-\psi' + \phi'\cos(\theta)\mathbf{k}$$

Thus, $\omega_x = 0$ $\omega_y = \phi'\sin(\theta)$ $\omega_z = -\psi' + \phi'\cos(\theta)$

$$\omega' = \phi' \times \psi' = -\phi'\,\psi'\sin(\theta)$$

$$\omega'_x = -\phi'\,\psi'\sin(\theta) \qquad \omega'_y = \omega'_z = 0$$

Applying

$$\Sigma M_x = I_x\omega'_x + \left(I_z - I_y\right)\omega_z\omega_y$$

$$F\,r\sin(\theta) - N\,r\cos(\theta) = \frac{mr^2}{2}\left(-\phi'\,\psi'\sin(\theta)\right) + \left(mr^2 - \frac{mr^2}{2}\right)\left(-\psi' + \phi'\cos(\theta)\right)\left(\phi'\sin(\theta)\right)$$

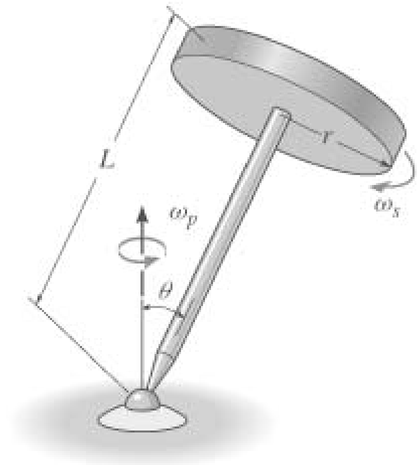
Solving we find

$$m\,a\phi'^2 r\sin(\theta) - m\,g\,r\cos(\theta) = \left(\frac{-mr^2}{2}\right)\phi'^2\sin(\theta)\left(\frac{a + r\cos(\theta)}{r}\right) - \left(\frac{mr^2}{2}\right)\left(\frac{a}{r}\right)\phi'^2\sin(\theta)$$

$$2g\cos(\theta) = a\phi'^2\sin(\theta) + r\phi'^2\sin(\theta)\cos(\theta) \qquad \phi' = \sqrt{\frac{2g\cot(\theta)}{a + r\cos(\theta)}}$$

Problem 21-70

The top consists of a thin disk that has weight W and radius r. The rod has a negligible mass and length L. If the top is spinning with an angular velocity ω_s, determine the steady-state precessional angular velocity ω_p.



Given:

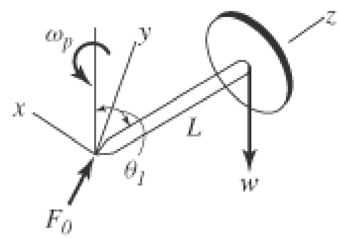
$W = 8 \text{ lb}$ $\theta = 40 \text{ deg}$

$r = 0.3 \text{ ft}$ $g = 32.2 \dfrac{\text{ft}}{\text{s}^2}$

$L = 0.5 \text{ ft}$ $\omega_s = 300 \dfrac{\text{rad}}{\text{s}}$

Solution:

$$\Sigma M_x = -I\,\phi'^2 \sin(\theta)\cos(\theta) + I_z\phi' \sin(\theta)(\phi' \cos(\theta) + \psi')$$



Guess $\omega_p = 1 \dfrac{\text{rad}}{\text{s}}$ Given

$$W\,L\sin(\theta) = -\left[\left(\frac{W}{g}\right)\left(\frac{r^2}{4}\right) + \left(\frac{W}{g}\right)L^2\right]\omega_p^{\,2}\sin(\theta)\cos(\theta) + \left(\frac{W}{g}\right)\left(\frac{r^2}{2}\right)\omega_p \sin(\theta)(\omega_p \cos(\theta) + \omega_s)$$

$\omega_p = \text{Find}(\omega_p)$ $\omega_p = 1.21 \dfrac{\text{rad}}{\text{s}}$ low precession

Guess $\omega_p = 70 \dfrac{\text{rad}}{\text{s}}$ Given

$$W\,L\sin(\theta) = -\left[\left(\frac{W}{g}\right)\left(\frac{r^2}{4}\right) + \left(\frac{W}{g}\right)L^2\right]\omega_p^{\,2}\sin(\theta)\cos(\theta) + \left(\frac{W}{g}\right)\left(\frac{r^2}{2}\right)\omega_p \sin(\theta)(\omega_p \cos(\theta) + \omega_s)$$

$\omega_p = \text{Find}(\omega_p)$ $\omega_p = 76.3 \dfrac{\text{rad}}{\text{s}}$ high precession

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Problem 21-71

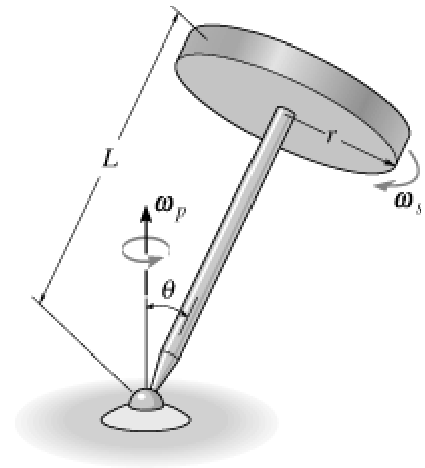
The top consists of a thin disk that has weight W and radius r. The rod has a negligible mass and length L. If the top is spinning with an angular velocity ω_s, determine the steady-state precessional angular velocity ω_p.

Given:

$W = 8\ \text{lb}$ $\theta = 90\ \text{deg}$

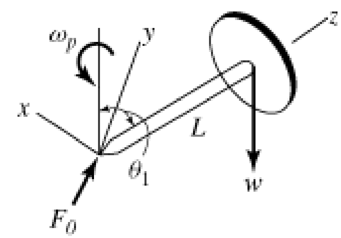
$r = 0.3\ \text{ft}$ $g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

$L = 0.5\ \text{ft}$ $\omega_s = 300\ \dfrac{\text{rad}}{\text{s}}$



Solution:

$$\Sigma M_x = -I\,\phi'^2 \sin(\theta)\cos(\theta) + I_z\phi' \sin(\theta)(\phi' \cos(\theta) + \psi')$$



Guess $\omega_p = 1\ \dfrac{\text{rad}}{\text{s}}$ Given

$$W L \sin(\theta) = -\left[\left(\frac{W}{g}\right)\left(\frac{r^2}{4}\right) + \left(\frac{W}{g}\right)L^2\right]\omega_p{}^2 \sin(\theta)\cos(\theta) + \left(\frac{W}{g}\right)\left(\frac{r^2}{2}\right)\omega_p \sin(\theta)(\omega_p \cos(\theta) + \omega_s)$$

$\omega_p = \text{Find}(\omega_p)$ $\omega_p = 1.19\ \dfrac{\text{rad}}{\text{s}}$

***Problem 21-72**

The top has weight *W* and can be considered as a solid cone. If it is observed to precess about the vertical axis at a constant rate of ω_y, determine its spin ω_s.

Given:

$W = 3\ \text{lb}$

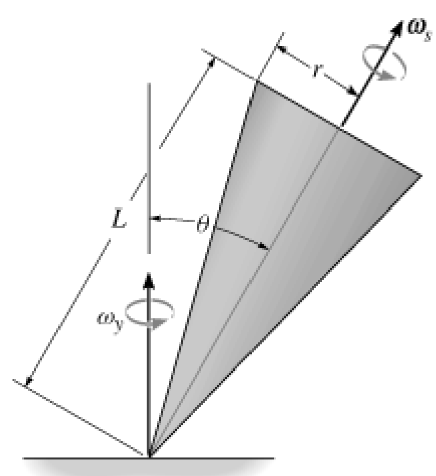
$\omega_y = 5\ \dfrac{\text{rad}}{\text{s}}$

$\theta = 30\ \text{deg}$

$L = 6\ \text{in}$

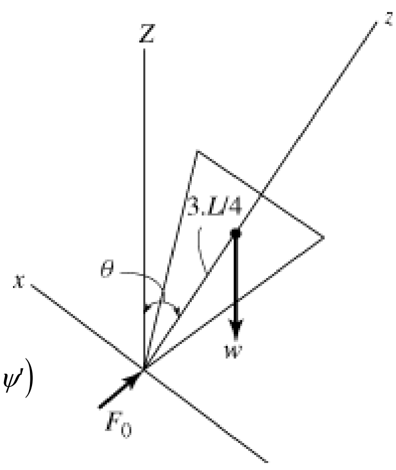
$r = 1.5\ \text{in}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution:

$$I = \frac{3}{80}\left(\frac{W}{g}\right)\left(4r^2 + L^2\right) + \left(\frac{W}{g}\right)\left(\frac{3L}{4}\right)^2$$



$$I_z = \frac{3}{10}\left(\frac{W}{g}\right)r^2$$

$$\Sigma M_x = -I\,\phi'^2 \sin(\theta)\cos(\theta) + I_z\phi' \sin(\theta)\left(\phi' \cos(\theta) + \psi'\right)$$

$$W\frac{3L}{4}\sin(\theta) = -I\,\omega_y{}^2 \sin(\theta)\cos(\theta) + I_z\omega_y \sin(\theta)\left(\omega_y \cos(\theta) + \psi'\right)$$

$$\psi' = \frac{1}{4}\left(\frac{3\,WL + 4\,I\,\omega_y{}^2\cos(\theta) - 4\,I_z\,\omega_y{}^2\cos(\theta)}{I_z\omega_y}\right)$$

$$\psi' = 652\,\frac{\text{rad}}{\text{s}}$$

Problem 21-73

The toy gyroscope consists of a rotor R which is attached to the frame of negligible mass. If it is observed that the frame is precessing about the pivot point O at rate ω_p determine the angular velocity ω_R of the rotor. The stem OA moves in the horizontal plane. The rotor has mass M and a radius of gyration k_{OA} about OA.

Given:

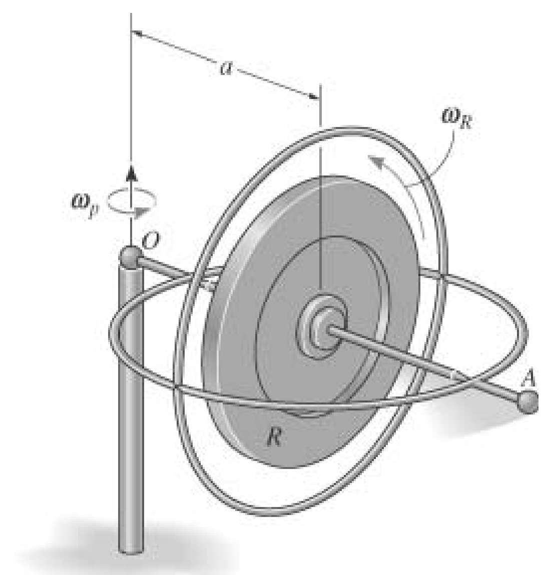
$$\omega_p = 2\,\frac{\text{rad}}{\text{s}}$$

$$M = 200\ \text{gm}$$

$$k_{OA} = 20\ \text{mm}$$

$$a = 30\ \text{mm}$$

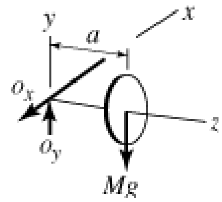
$$g = 9.81\,\frac{\text{m}}{\text{s}^2}$$



Solution:

$$\Sigma M_x = I_z\Omega_y\omega_z$$

$$M g a = M k_{OA}^2 \omega_p \omega_R$$



$$\omega_R = \frac{g a}{k_{OA}^2 \omega_p}$$

$$\omega_R = 368 \frac{\text{rad}}{\text{s}}$$

Problem 21-74

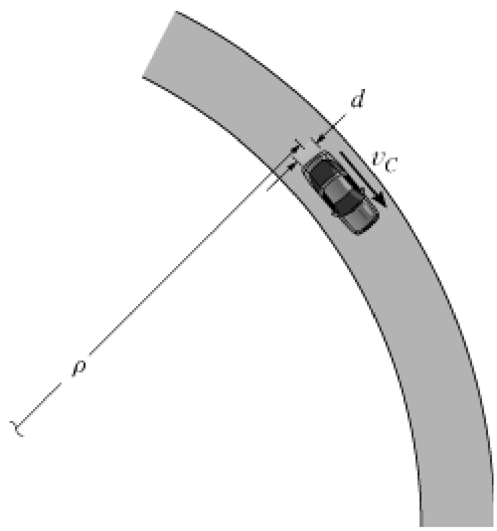
The car is traveling at velocity v_c around the horizontal curve having radius ρ. If each wheel has mass M, radius of gyration k_G about its spinning axis, and radius r, determine the difference between the normal forces of the rear wheels, caused by the gyroscopic effect. The distance between the wheels is d.

Given:

$v_c = 100 \frac{\text{km}}{\text{hr}}$ $\quad k_G = 300 \text{ mm}$

$\rho = 80 \text{ m}$ $\quad r = 400 \text{ mm}$

$M = 16 \text{ kg}$ $\quad d = 1.3 \text{ m}$



Solution:

$I = 2M k_G^2$ $\quad I = 2.88 \text{ kg}\cdot\text{m}^2$

$\omega_s = \frac{v_c}{r}$ $\quad \omega_s = 69.44 \frac{\text{rad}}{\text{s}}$

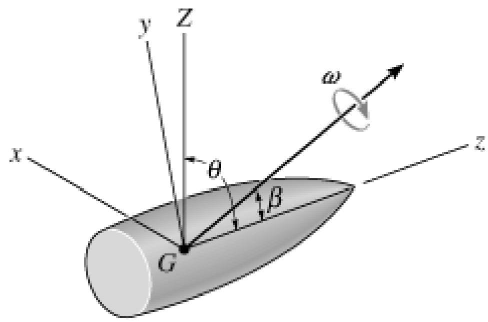
$\omega_p = \frac{v_c}{\rho}$ $\quad \omega_p = 0.35 \frac{\text{rad}}{\text{s}}$

$M = I\omega_s\omega_p$

$\Delta F\, d = I\omega_s\omega_p$ $\quad \Delta F = I\,\omega_s \frac{\omega_p}{d}$ $\quad \Delta F = 53.4 \text{ N}$

Problem 21-75

The projectile shown is subjected to torque-free motion. The transverse and axial moments of inertia are I and I_z respectively. If θ represents the angle between the precessional axis Z and the axis of symmetry z, and β is the angle between the angular velocity ω and the z axis, show that β and θ are related by the equation $\tan\theta = (I/I_z)\tan\beta$.



Solution:

From Eq. 21-34 $\quad \omega_y = \dfrac{H_G \sin(\theta)}{I}$ and $\omega_z = \dfrac{H_G \cos(\theta)}{I_z}$

Hence $\quad \dfrac{\omega_y}{\omega_z} = \dfrac{I_z}{I}\tan(\theta)$

However, $\quad \omega_y = \omega \sin(\beta)$ and $\omega_z = \omega \cos(\beta)$

$$\frac{\omega_y}{\omega_z} = \tan(\beta) = \frac{I_z}{I}\tan(\theta)$$

$$\tan(\theta) = \frac{I}{I_z}\tan(\beta)$$ Q.E.D

***Problem 21-76**

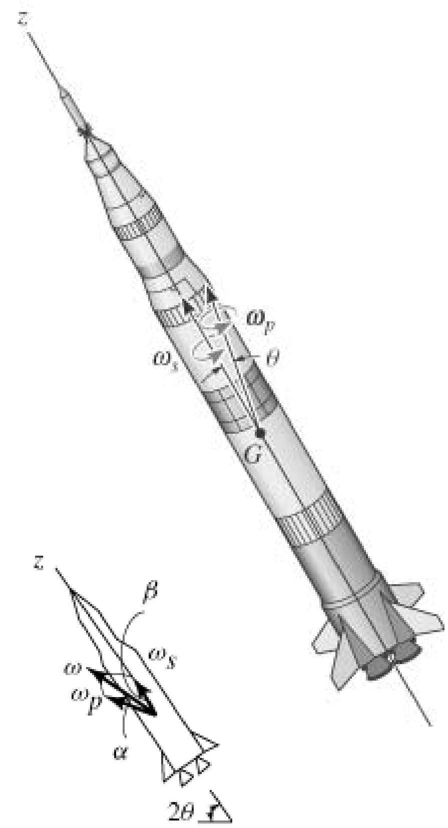
While the rocket is in free flight, it has a spin ω_s and precesses about an axis measured angle θ from the axis of spin. If the ratio of the axial to transverse moments of inertia of the rocket is r, computed about axes which pass through the mass center G, determine the angle which the resultant angular velocity makes with the spin axis. Construct the body and space cones used to describe the motion. Is the precession regular or retrograde?

Given:

$\omega_s = 3\ \dfrac{\text{rad}}{\text{s}}$

$\theta = 10\ \text{deg}$

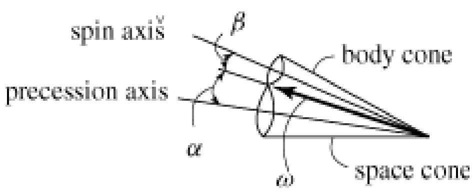
$$r = \frac{1}{15}$$



Solution:

Determine the angle β from the result of prob.21-75

$$\tan(\theta) = \frac{\tan(\beta)}{r}$$



$\beta = \operatorname{atan}(r\tan(\theta))$ $\qquad$ $\beta = 0.673\ \text{deg}$

Thus,

$\alpha = \theta - \beta$ $\qquad$ $\alpha = 9.33\ \text{deg}$

Regular Precession $\quad$ Since $\quad I_z < I$

Problem 21-77

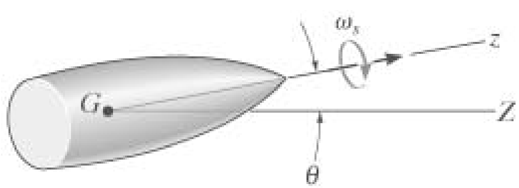
The projectile has a mass M and axial and transverse radii of gyration k_z and k_t, respectively. If it is spinning at ω_s when it leaves the barrel of a gun, determine its angular momentum. Precession occurs about the Z axis.

Given:

$M = 0.9\ \text{kg}$ $\qquad$ $\omega_s = 6\ \frac{\text{rad}}{\text{s}}$

$k_z = 20\ \text{mm}$ $\qquad$ $\theta = 10\ \text{deg}$

$k_t = 25\ \text{mm}$



Solution:

$I = M k_t^2$ $\qquad$ $I = 5.625 \times 10^{-4}\ \text{kg}\cdot\text{m}^2$

$I_z = M k_z^2$ $\qquad$ $I_z = 3.600 \times 10^{-4}\ \text{kg}\cdot\text{m}^2$

$\psi = \omega_s$

$$\psi = \left(\frac{I - I_z}{I\,I_z}\right)H_G \cos(\theta)$$

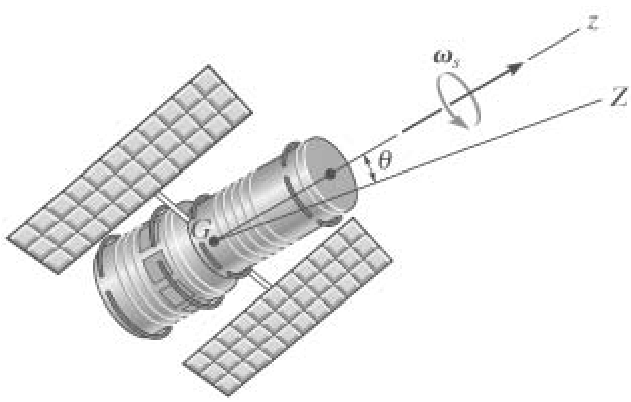
$$H_G = \psi I\left[\frac{I_z}{\cos(\theta)(I - I_z)}\right] \qquad H_G = 6.09 \times 10^{-3}\ \frac{\mathrm{kg \cdot m^2}}{\mathrm{s}}$$

Problem 21-78

The satellite has mass M, and about axes passing through the mass center G the axial and transverse radii of gyration are k_z and k_t, respectively. If it is spinning at ω_s when it is launched, determine its angular momentum. Precession occurs about the Z axis.

Units Used:

$\mathrm{Mg} = 10^3\ \mathrm{kg}$



Given:

$M = 1.8\ \mathrm{Mg}$ $\qquad \omega_s = 6\ \frac{\mathrm{rad}}{\mathrm{s}}$

$k_z = 0.8\ \mathrm{m}$

$k_t = 1.2\ \mathrm{m}$ $\qquad \theta = 5\ \mathrm{deg}$

Solution:

$$I = M k_t^2 \qquad I = 2592\,\mathrm{kg \cdot m^2}$$

$$I_z = M k_z^2 \qquad I_z = 1152\,\mathrm{kg \cdot m^2}$$

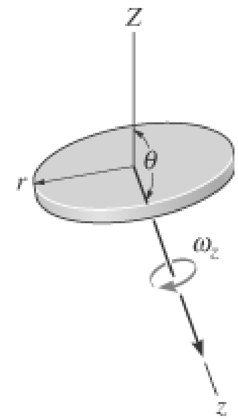
$$\psi' = \omega_s$$

$$\psi' = \left(\frac{I - I_z}{I\,I_z}\right)H_G \cos(\theta)$$

$$H_G = \psi' I\left[\frac{I_z}{\cos(\theta)(I - I_z)}\right] \qquad H_G = 12.5\,\mathrm{Mg} \cdot \frac{\mathrm{m^2}}{\mathrm{s}}$$

Problem 21-79

The disk of mass M is thrown with a spin ω_z. The angle θ is measured as shown. Determine the precession about the Z axis.



Given: $M = 4\ \text{kg}$

$\theta = 160\ \text{deg}$

$r = 125\ \text{mm}$

$\omega_z = 6\ \frac{\text{rad}}{\text{s}}$

Solution:

$$I = \frac{1}{4}M r^2 \qquad I_z = \frac{1}{2}M r^2$$

Applying Eq.21 - 36

$$\psi' = \omega_z = \frac{I - I_z}{I I_z} H_G \cos(\theta)$$

$$H_G = \omega_z \frac{I I_z}{\cos(\theta)(I - I_z)} \qquad H_G = 0.1995\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

$$\phi' = \frac{H_G}{I} \qquad \phi' = 12.8\ \frac{\text{rad}}{\text{s}}$$

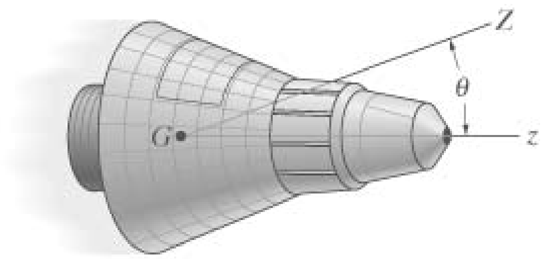
Note that this is a case of retrograde precession since $I_z > I$

***Problem 21-80**

The radius of gyration about an axis passing through the axis of symmetry of the space capsule of mass M is k_z, and about any transverse axis passing through the center of mass G, is k_t. If the capsule has a known steady-state precession of two revolutions per hour about the Z axis, determine the rate of spin about the z axis.

Units Used:

$\text{Mg} = 10^3\ \text{kg}$



Given:

$M = 1.6\ \text{Mg}$

$k_z = 1.2\ \text{m}$

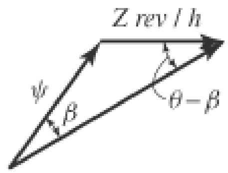
$k_t = 1.8\ \text{m}$

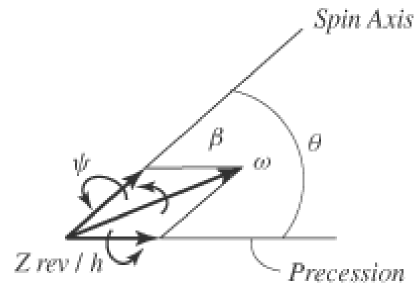
$\theta = 20\ \text{deg}$

Solution:

$I = M k_t^2$

$I_z = M k_z^2$





Using the Eqn.

$$\tan(\theta) = \left(\frac{I}{I_z}\right)\tan(\beta)$$

$$\beta = \operatorname{atan}\left(\tan(\theta)\frac{I_z}{I}\right)$$

$$\beta = 9.19\ \text{deg}$$