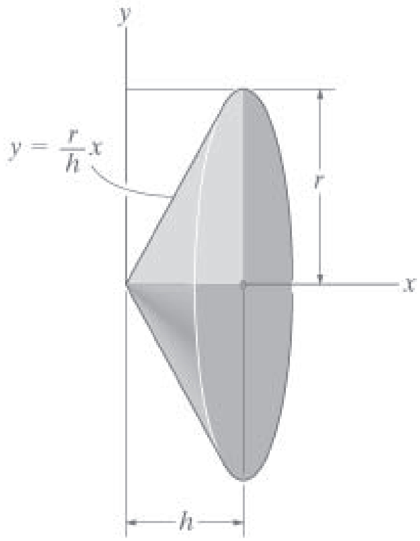
Problem 17-1

The right circular cone is formed by revolving the shaded area around the x axis. Determine the moment of inertia I_x and express the result in terms of the total mass m of the cone. The cone has a constant density ρ.



Solution:

$$m = \int_0^h \rho\,\pi\left(\frac{rx}{h}\right)^2 \mathrm{d}x = \frac{1}{3}\,h\,\rho\,\pi r^2$$

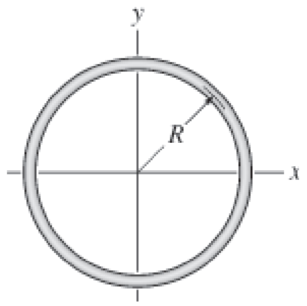
$$\rho = \frac{3m}{h\pi r^2}$$

$$I_x = \frac{3m}{h\pi r^2}\int_0^h \frac{1}{2}\pi\left(\frac{rx}{h}\right)^2\left(\frac{rx}{h}\right)^2 \mathrm{d}x \qquad \boxed{I_x = \frac{3}{10}m\,r^2}$$

Problem 17-2

Determine the moment of inertia of the thin ring about the z axis. The ring has a mass m.

Solution:



$$m = \int_0^{2\pi} \rho R\,\mathrm{d}\theta = 2\,\pi\,\rho R \qquad \rho = \frac{m}{2\pi R}$$

$$I_z = \frac{m}{2\pi R}\int_0^{2\pi} R\,R^2\,\mathrm{d}\theta = m\,R^2 \qquad \boxed{I_z = m\,R^2}$$

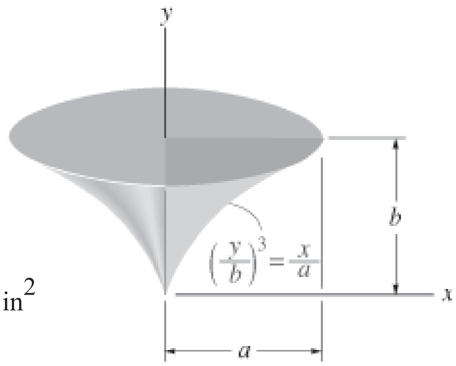
Problem 17-3

The solid is formed by revolving the shaded area around the y axis. Determine the radius of gyration k_y. The specific weight of the material is γ.

Given:

$$\gamma = 380\,\frac{\mathrm{lb}}{\mathrm{ft}^3} \qquad a = 3\text{ in} \qquad b = 3\text{ in}$$

Solution:



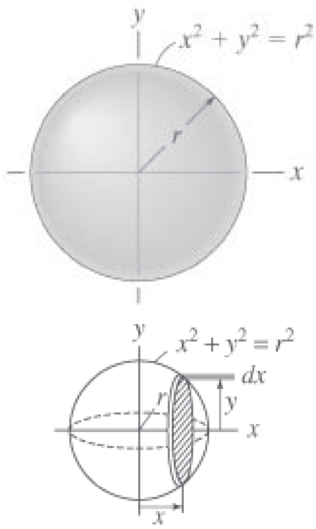
$$m = \int_0^b \gamma\pi\left(a\frac{y^3}{b^3}\right)^2 dy \qquad m = 0.083 \text{ slug}$$

$$I_y = \int_0^b \gamma\frac{1}{2}\pi\left(a\frac{y^3}{b^3}\right)^2\left(a\frac{y^3}{b^3}\right)^2 dy \qquad I_y = 0.201 \text{ slug}\cdot\text{in}^2$$

$$k = \sqrt{\frac{I_y}{m}} \qquad k = 1.56 \text{ in}$$

***Problem 17-4**

Determine the moment of inertia I_x of the sphere and express the result in terms of the total mass m of the sphere.The sphere has a constant density ρ.



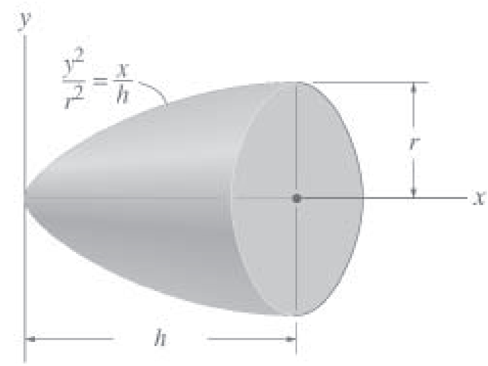
Solution:

$$m = \int_{-r}^{r} \rho\pi\left(r^2 - x^2\right) dx = \frac{4}{3}r^3\rho\pi \qquad \rho = \frac{3m}{4\pi r^3}$$

$$I_x = \frac{3m}{4\pi r^3}\int_{-r}^{r} \frac{\pi}{2}\left(r^2 - x^2\right)^2 dx \qquad I_x = \frac{2}{5}m r^2$$

Problem 17-5

Determine the radius of gyration k_x of the paraboloid. The density of the material is ρ.



Units Used: $\quad \text{Mg} = 10^6 \text{ gm}$

Given:

$h = 200 \text{ mm}$

$r = 100 \text{ mm}$

$\rho = 5 \frac{\text{Mg}}{\text{m}^3}$

Solution:

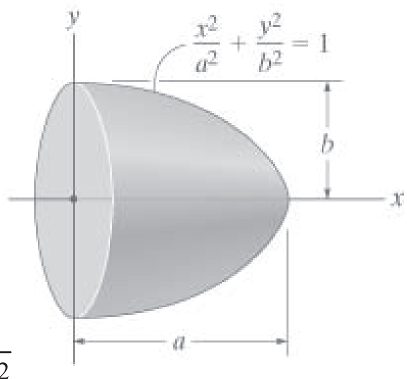
$$M = \int_0^h \rho\,\pi\left(\frac{x\,r^2}{h}\right) dx \qquad M = 15.708\ \text{kg}$$

$$I_x = \int_0^h \frac{1}{2}\rho\,\pi\left(\frac{x\,r^2}{h}\right)^2 dx \qquad I_x = 0.052\ \text{kg}\cdot\text{m}^2$$

$$k_x = \sqrt{\frac{I_x}{M}} \qquad k_x = 57.7\,\text{mm}$$

Problem 17-6

Determine the moment of inertia of the semiellipsoid with respect to the x axis and express the result in terms of the mass m of the semiellipsoid. The material has a constant density ρ.



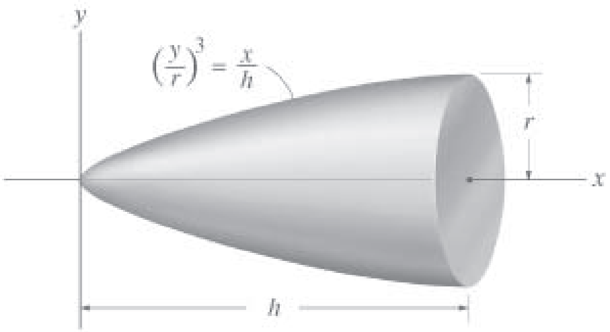
Solution:

$$m = \int_0^a \rho\,\pi b^2\left(1 - \frac{x^2}{a^2}\right) dx = \frac{2}{3}a\,\rho\,\pi b^2 \qquad \rho = \frac{3m}{2a\pi b^2}$$

$$I_x = \frac{3m}{2a\pi b^2}\int_0^a \frac{1}{2}\pi\left[b^2\left(1 - \frac{x^2}{a^2}\right)\right]^2 dx \qquad I_x = \frac{2}{5}m\,b^2$$

Problem 17-7

Determine the radius of gyration k_x of the body. The specific weight of the material is γ.



Given:

$$\gamma = 380\ \frac{\text{lb}}{\text{ft}^3}$$

$$h = 8\ \text{in}$$

$$r = 2\ \text{in}$$

Solution:

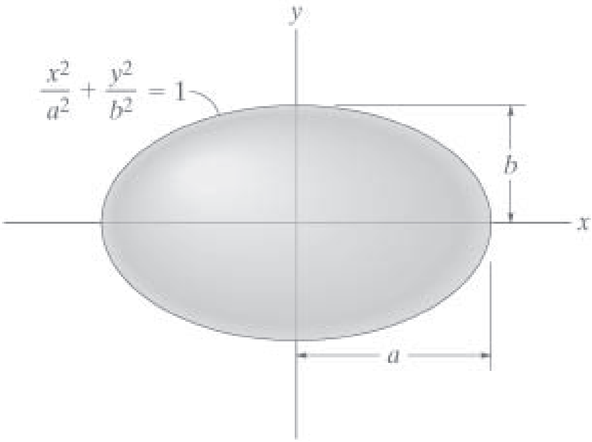
$$M = \int_0^h \gamma \pi \left[r\left(\frac{x}{h}\right)^{\frac{1}{3}}\right]^2 dx \qquad M = 0.412\ \text{slug}$$

$$I_x = \int_0^h \frac{1}{2}\gamma \pi \left[r\left(\frac{x}{h}\right)^{\frac{1}{3}}\right]^4 dx \qquad I_x = 0.589\ \text{slug}\cdot\text{in}^2$$

$$k_x = \sqrt{\frac{I_x}{M}} \qquad k_x = 1.20\ \text{in}$$

***Problem 17-8**

Determine the moment of inertia of the ellipsoid with respect to the x axis and express the result in terms of the mass m of the ellipsoid. The material has a constant density ρ.



Solution:

$$m = \int_{-a}^{a} \rho \pi b^2\left(1 - \frac{x^2}{a^2}\right) dx = \frac{4}{3} a \rho \pi b^2$$

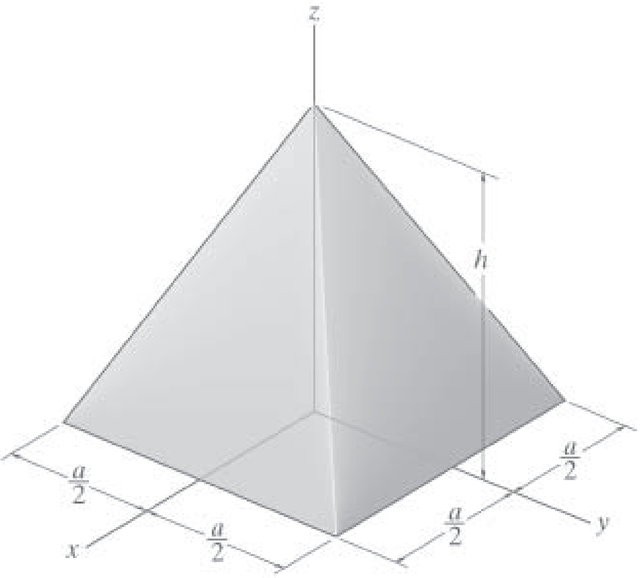
$$\rho = \frac{3m}{4a\pi b^2}$$

$$I_x = \frac{3m}{4a\pi b^2}\int_{-a}^{a} \frac{1}{2}\pi \left[b^2\left(1 - \frac{x^2}{a^2}\right)\right]^2 dx \qquad I_x = \frac{2}{5} m b^2$$

Problem 17-9

Determine the moment of inertia of the homogeneous pyramid of mass m with respect to the z axis.The density of the material is ρ. *Suggestion*: Use a rectangular plate element having a volume of $dV = (2x)(2y)\,dz$.

Solution:



$$x = \frac{a(h-z)}{2h}$$

$$m = \int_0^h \rho\left[\frac{a(h-z)}{h}\right]^2 dz = \frac{1}{3}h\rho a^2$$

$$\rho = \frac{3m}{h a^2}$$

$$I_z = \frac{3m}{h a^2}\int_0^h \frac{1}{6}\left[\frac{a(h-z)}{h}\right]^4 dz$$

$$\boxed{I_z = \frac{1}{10}m a^2}$$

Problem 17-10

The concrete shape is formed by rotating the shaded area about the y axis. Determine the moment of inertia I_y. The specific weight of concrete is γ.

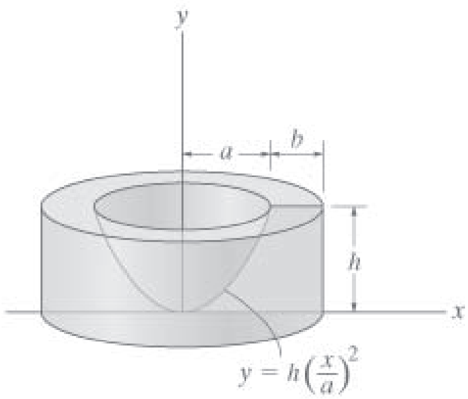
Given:

$a = 6$ in

$b = 4$ in

$h = 8$ in

$\gamma = 150 \frac{\text{lb}}{\text{ft}^3}$



Solution:

$$I_y = \int_0^h \gamma\left[\frac{1}{2}\pi(a+b)^4 - \frac{1}{2}\pi\left(a^2\frac{y}{h}\right)^2\right] dy$$

$$\boxed{I_y = 2.25 \text{ slug}\cdot\text{ft}^2}$$

Problem 17-11

Determine the moment of inertia of the thin plate about an axis perpendicular to the page and passing through the pin at O. The plate has a hole in its center. Its thickness is t, and the material has a density of ρ.

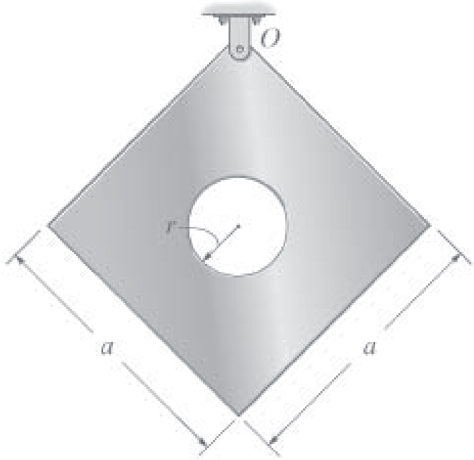
Given:

$a = 1.40\ \text{m}$

$r = 150\ \text{mm}$

$t = 50\ \text{mm}$

$\rho = 50\ \frac{\text{kg}}{\text{m}^3}$



Solution:

$$I_O = \rho a^2 t\left(\frac{2a^2}{3}\right) - \left[\rho\,\pi r^2 t\left(\frac{r^2}{2}\right) + \rho\,\pi r^2 t\left(\frac{\sqrt{2}\,a}{2}\right)^2\right]$$

$I_O = 6.227\ \text{kg}\cdot\text{m}^2$

***Problem 17-12**

Determine the moment of inertia I_z of the frustum of the cone which has a conical depression. The material has a density ρ.

Given:

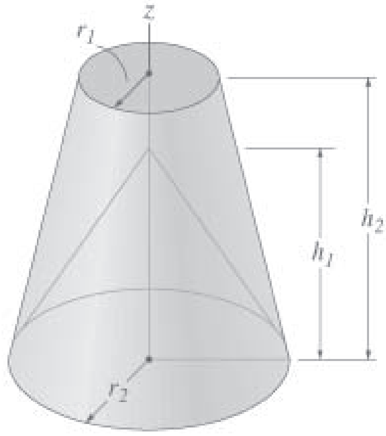
$r_1 = 0.2\ \text{m}$

$r_2 = 0.4\ \text{m}$

$h_1 = 0.6\ \text{m}$

$h_2 = 0.8\ \text{m}$

$\rho = 200\ \frac{\text{kg}}{\text{m}^3}$



Solution:

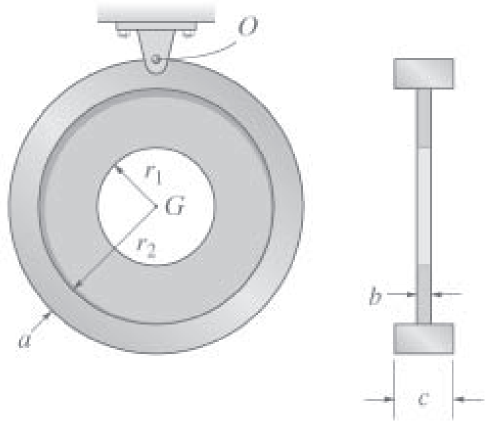
$$h_3 = \frac{r_2 h_2}{r_2 - r_1} \qquad h_4 = h_3 - h_2$$

$$I_z = \rho\left(\frac{\pi r_2^2 h_3}{3}\right)\left(\frac{3}{10}\right)r_2^2 - \left(\frac{\rho\,\pi r_1^2 h_4}{3}\right)\left(\frac{3}{10}\right)r_1^2 - \left(\frac{\rho\,\pi r_2^2 h_1}{3}\right)\left(\frac{3}{10}\right)r_2^2$$

$I_z = 1.53\ \text{kg}\cdot\text{m}^2$

Problem 17-13

Determine the moment of inertia of the assembly about an axis which is perpendicular to the page and passes through the center of mass G. The material has a specific weight γ.



Given:

$a = 0.5\ \text{ft}$ $\quad r_1 = 1\ \text{ft}$

$b = 0.25\ \text{ft}$ $\quad r_2 = 2\ \text{ft}$

$c = 1\ \text{ft}$ $\quad \gamma = 90\ \dfrac{\text{lb}}{\text{ft}^3}$

Solution:

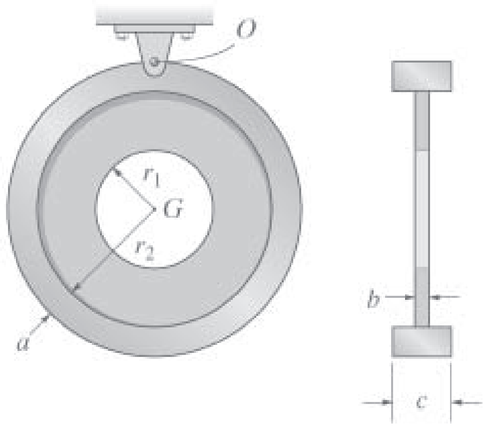
$$I_G = \frac{1}{2}\gamma\pi c\left(r_2 + a\right)^4 - \frac{1}{2}\gamma\pi(c - b)\, r_2^{\,4} - \frac{1}{2}\gamma\pi b\, r_1^{\,4}$$

$I_G = 118\ \text{slug}\cdot\text{ft}^2$

Problem 17-14

Determine the moment of inertia of the assembly about an axis which is perpendicular to the page and passes through point O. The material has a specific weight γ.

Given:



$a = 0.5\ \text{ft}$

$b = 0.25\ \text{ft}$

$c = 1\ \text{ft}$

$r_1 = 1\ \text{ft}$

$r_2 = 2\ \text{ft}$

$\gamma = 90\ \dfrac{\text{lb}}{\text{ft}^3}$

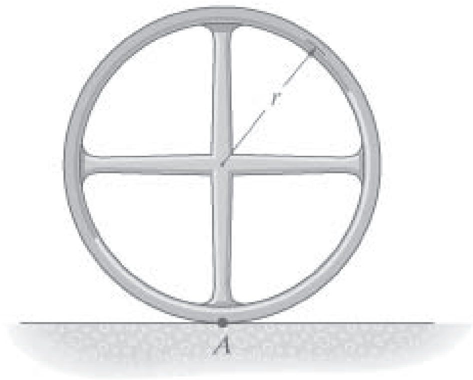
Solution:

$$I_O = \frac{3}{2}\gamma\pi c\left(r_2 + a\right)^4 - \left[\frac{1}{2}\gamma\pi(c - b)\, r_2^{\,4} + \gamma\pi(c - b)\, r_2^{\,2}\left(r_2 + a\right)^2\right] \ldots$$
$$+ -\left[\frac{1}{2}\gamma\pi b\, r_1^{\,4} + \gamma\pi b\, r_1^{\,2}\left(r_2 + a\right)^2\right]$$

$I_O = 283\ \text{slug}\cdot\text{ft}^2$

Problem 17-15

The wheel consists of a thin ring having a mass M_1 and four spokes made from slender rods, each having a mass M_2. Determine the wheel's moment of inertia about an axis perpendicular to the page and passing through point A.



Given:

$M_1 = 10$ kg

$M_2 = 2$ kg

$r = 500$ mm

Solution:

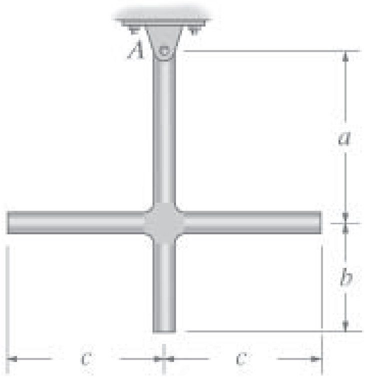
$$I_G = M_1 r^2 + 4M_2\left(\frac{r^2}{3}\right)$$

$$I_A = I_G + \left(M_1 + 4M_2\right)r^2$$

$$I_A = 7.67\ \text{kg}\cdot\text{m}^2$$

Problem 17-16

The slender rods have a weight density γ. Determine the moment of inertia of the assembly about an axis perpendicular to the page and passing through point A.



Given:

$\gamma = 3\ \frac{\text{lb}}{\text{ft}}$

$a = 2$ ft

$b = 1$ ft

$c = 1.5$ ft

Solution:

$$I_A = \gamma(a+b)\left[\frac{(a+b)^2}{3}\right] + \gamma(2c)\frac{(2c)^2}{12} + \gamma(2c)\,a^2$$

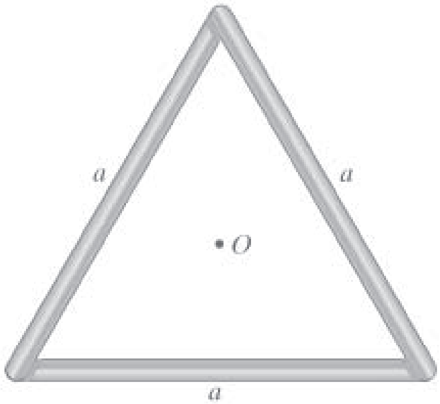
$$I_A = 2.17\ \text{slug}\cdot\text{ft}^2$$

Problem 17-17

Each of the three rods has a mass m. Determine the moment of inertia of the assembly about an axis which is perpendicular to the page and passes through the center point O.

Solution:

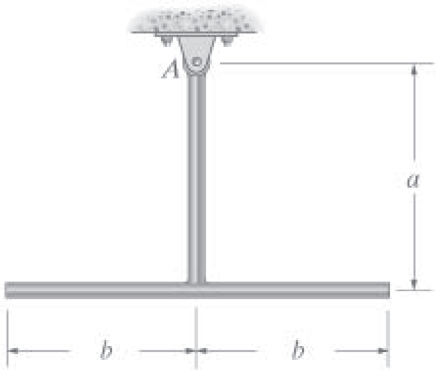
$$I_0 = 3\left[\frac{1}{12} m a^2 + m\left[\frac{(a)\sin(60\ \text{deg})}{3}\right]^2\right]$$



$$I_0 = \frac{1}{2} m a^2$$

Problem 17-18

The slender rods have weight density γ. Determine the moment of inertia of the assembly about an axis perpendicular to the page and passing through the pin at A.



Given:

$\gamma = 3\ \frac{\text{lb}}{\text{ft}}$

$a = 2\ \text{ft}$

$b = 1.5\ \text{ft}$

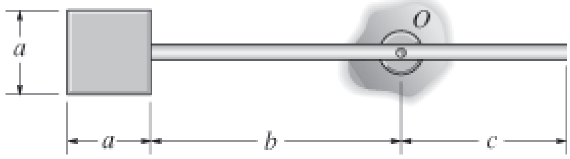
Solution:

$$I_A = \frac{1}{3}\gamma a^3 + \frac{1}{12}\gamma(2b)^3 + \gamma(2b)\,a^2$$

$$I_A = 1.58\ \text{slug}\cdot\text{ft}^2$$

Problem 17-19

The pendulum consists of a plate having weight W_p and a slender rod having weight W_r. Determine the radius of gyration of the pendulum about an axis perpendicular to the page and passing through point O.



Given:

$W_p = 12\ \text{lb}$

$W_r = 4\ \text{lb}$

$a = 1\ \text{ft}$

$b = 3\ \text{ft}$

$c = 2\ \text{ft}$

Solution:

$$I_O = \frac{1}{3}\left(\frac{b}{b+c}\right)W_r b^2 + \frac{1}{3}\left(\frac{c}{b+c}\right)W_r c^2 + \frac{1}{6}W_p a^2 + W_p\left(b+\frac{a}{2}\right)^2$$

$$I_O = 4.921\ \text{slug}\cdot\text{ft}^2$$

$$k_O = \sqrt{\frac{I_O}{W_r + W_p}} \qquad k_O = 3.146\ \text{ft}$$

***Problem 17-20**

Determine the moment of inertia of the overhung crank about the x axis. The material is steel having a density ρ.

Units Used:

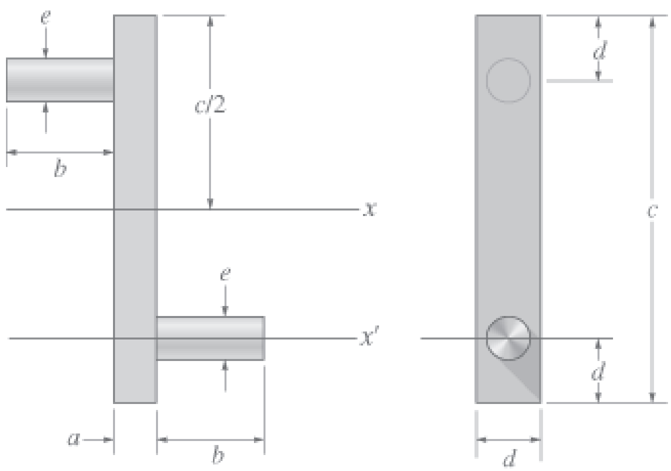
$$\text{Mg} = 10^3\ \text{kg}$$

Given:

$$\rho = 7.85\ \frac{\text{Mg}}{\text{m}^3} \qquad c = 180\ \text{mm}$$

$$a = 20\ \text{mm} \qquad d = 30\ \text{mm}$$

$$b = 50\ \text{mm} \qquad e = 20\ \text{mm}$$



Solution:

$$I_x = 2\left[\frac{\rho\pi}{2}\left(\frac{e}{2}\right)^2 b\left(\frac{e}{2}\right)^2 + \rho\pi\left(\frac{e}{2}\right)^2 b\left(\frac{c-2d}{2}\right)^2\right] + \frac{\rho a d c}{12}\left(d^2 + c^2\right)$$

$$I_x = 3.25 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

Problem 17-21

Determine the moment of inertia of the overhung crank about the x' axis. The material is steel having a density ρ.

Units Used:

$$\text{Mg} = 10^3\ \text{kg}$$

Given:

$\rho = 7.85 \frac{\text{Mg}}{\text{m}^3}$

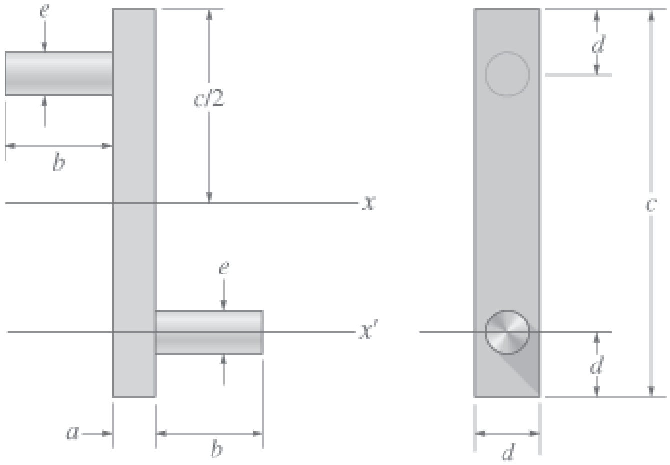
$a = 20$ mm

$b = 50$ mm

$c = 180$ mm

$d = 30$ mm

$e = 20$ mm



Solution:

$$I_x = 2\left[\frac{\rho\pi}{2}\left(\frac{e}{2}\right)^2 b\left(\frac{e}{2}\right)^2 + \rho\pi\left(\frac{e}{2}\right)^2 b\left(\frac{c-2d}{2}\right)^2\right] + \frac{\rho a d c}{12}\left(d^2 + c^2\right)$$

$$I_{x'} = I_x + \left[2\rho\pi\left(\frac{e}{2}\right)^2 b + \rho a d c\right]\left(\frac{c-2d}{2}\right)^2$$

$I_{x'} = 7.19 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$

Problem 17-22

Determine the moment of inertia of the solid steel assembly about the x axis. Steel has specific weight γ_{st}.

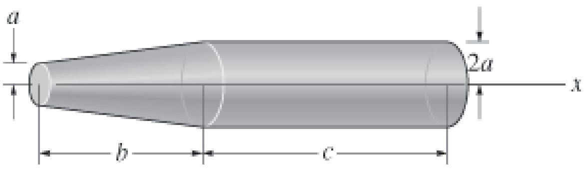
Given:

$a = 0.25$ ft

$b = 2$ ft

$c = 3$ ft

$\gamma_{st} = 490 \frac{\text{lb}}{\text{ft}^3}$

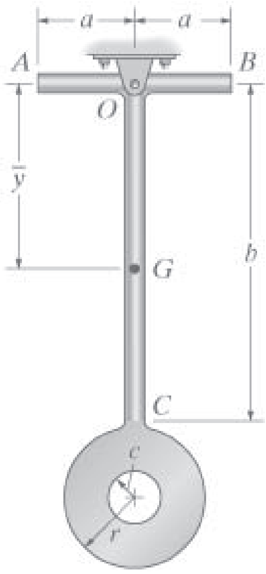


Solution:

$$I_x = \left[\frac{1}{2}\pi(2a)^2 c(2a)^2 + \frac{3}{10}\frac{1}{3}\pi(2a)^2(2b)(2a)^2 - \frac{3}{10}\frac{1}{3}\pi a^2 b a^2\right]\gamma_{st}$$

$I_x = 5.644\ \text{slug}\cdot\text{ft}^2$

Problem 17-23



The pendulum consists of two slender rods AB and OC which have a mass density ρ_1. The thin plate has a mass density ρ_2. Determine the location y' of the center of mass G of the pendulum, then calculate the moment of inertia of the pendulum about an axis perpendicular to the page and passing through G.

Given:

$$\rho_1 = 3\ \frac{\text{kg}}{\text{m}} \qquad a = 0.4\ \text{m} \qquad c = 0.1\ \text{m}$$

$$\rho_2 = 12\ \frac{\text{kg}}{\text{m}^2} \qquad b = 1.5\ \text{m} \qquad r = 0.3\ \text{m}$$

Solution:

$$y' = \frac{\rho_1 b\left(\frac{b}{2}\right) + \rho_2\pi\left(r^2 - c^2\right)(b + r)}{\rho_1(b + 2a) + \rho_2\pi\left(r^2 - c^2\right)} \qquad y' = 0.888\ \text{m}$$

$$I_O = \frac{1}{12}\rho_1(2a)^3 + \frac{1}{3}\rho_1 b^3 + \left(\frac{\rho_2}{2}\right)\pi r^4 + \rho_2\pi r^2(r + b)^2 - \left[\left(\frac{\rho_2}{2}\right)\pi c^4 + \rho_2\pi c^2(r + b)^2\right]$$

$$I_G = I_O - \left[\rho_1(2a + b) + \rho_2\pi\left(r^2 - c^2\right)\right]y'^2$$

$$I_G = 5.61\ \text{kg}\cdot\text{m}^2$$

***Problem 17-24**

Determine the greatest possible acceleration of the race car of mass M so that its front tires do not leave the ground or the tires slip on the track. The coefficients of static and kinetic friction are μ_s and μ_k respectively. Neglect the mass of the tires. The car has rear-wheel drive and the front tires are free to roll.

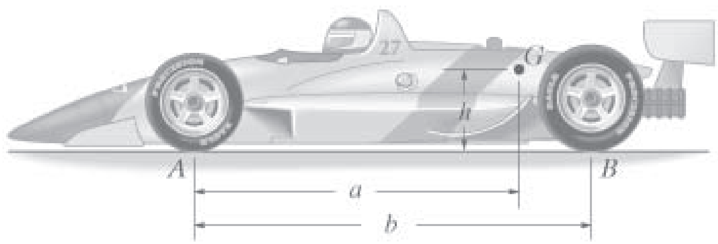
Given:

$$M = 975\ \text{kg} \qquad \mu_s = 0.8$$

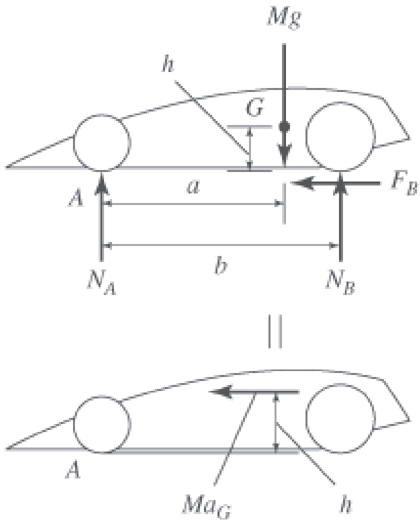
$$a = 1.82\ \text{m} \qquad \mu_k = 0.6$$

$$b = 2.20\ \text{m} \qquad g = 9.81\ \frac{\text{m}}{\text{s}^2}$$

$$h = 0.55\ \text{m}$$



Solution:



First assume that the rear wheels are on the verge of slipping $\qquad F_B = \mu_s N_B$

Guesses

$$N_A = 1\ \text{N} \qquad N_B = 1\ \text{N} \qquad a_G = 1\ \frac{\text{m}}{\text{s}^2}$$

Given

$$\mu_s N_B = M a_G \qquad N_A + N_B - M g = 0$$

$$-N_A a + N_B(b-a) - \mu_s N_B h = 0$$

$$\begin{pmatrix} N_A \\ N_B \\ a_{G1} \end{pmatrix} = \text{Find}(N_A, N_B, a_G) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} -326 \\ 9891 \end{pmatrix} \text{N} \qquad a_{G1} = 8.12\ \frac{\text{m}}{\text{s}^2}$$

Next assume that the front wheels lose contact with the ground $\qquad N_A = 0$

Guesses $\qquad N_B = 1\ \text{N} \qquad F_B = 1\ \text{N} \qquad a_G = 1\ \frac{\text{m}}{\text{s}^2}$

Given $\qquad F_B = M a_G \qquad N_B - M g = 0 \qquad N_B(b-a) - F_B h = 0$

$$\begin{pmatrix} N_B \\ F_B \\ a_{G2} \end{pmatrix} = \text{Find}(N_B, F_B, a_G) \qquad \begin{pmatrix} N_B \\ F_B \end{pmatrix} = \begin{pmatrix} 9565 \\ 6608 \end{pmatrix} \text{N} \qquad a_{G2} = 6.78\ \frac{\text{m}}{\text{s}^2}$$

Choose the critical case $\qquad a_G = \min(a_{G1}, a_{G2}) \qquad a_G = 6.78\ \frac{\text{m}}{\text{s}^2}$

Problem 17-25

Determine the greatest possible acceleration of the race car of mass M so that its front tires do not leave the ground nor the tires slip on the track. The coefficients of static and kinetic friction are μ_s and μ_k respectively. Neglect the mass of the tires. The car has four-wheel drive.

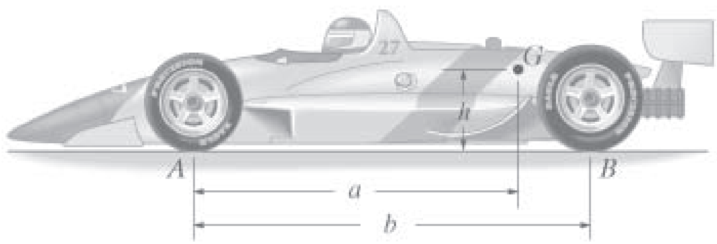
Given:

$M = 975\ \text{kg} \quad \mu_s = 0.8$

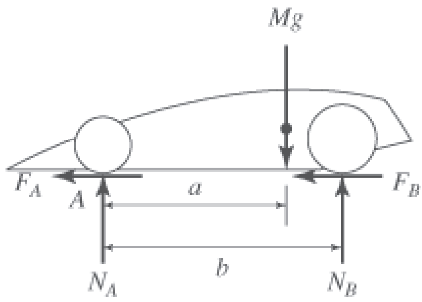
$a = 1.82\ \text{m} \quad \mu_k = 0.6$

$b = 2.20\ \text{m} \quad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$h = 0.55\ \text{m}$



Solution:



First assume that all wheels are on the verge of slipping $F_A = \mu_s N_A$

$F_B = \mu_s N_B$

Guesses

$N_A = 1\ \text{N} \qquad N_B = 1\ \text{N} \qquad a_G = 1\ \frac{\text{m}}{\text{s}^2}$

Given

$\mu_s N_B + \mu_s N_A = M a_G \qquad N_A + N_B - M g = 0$

$-N_A a + N_B(b-a) - \mu_s N_B h - \mu_s N_A h = 0$

$$\begin{pmatrix} N_A \\ N_B \\ a_{G1} \end{pmatrix} = \text{Find}(N_A, N_B, a_G) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} -261 \\ 9826 \end{pmatrix} \text{N} \qquad a_{G1} = 7.85\ \frac{\text{m}}{\text{s}^2}$$

Next assume that the front wheels lose contact with the ground $N_A = 0$

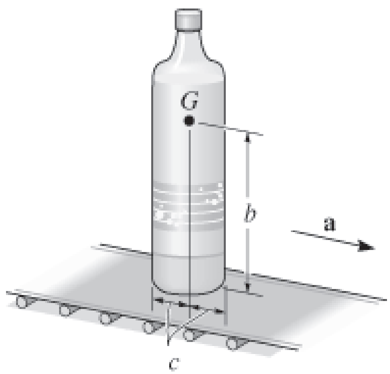
Guesses $N_B = 1\ \text{N} \qquad F_B = 1\ \text{N} \qquad a_G = 1\ \frac{\text{m}}{\text{s}^2}$

Given $F_B = M a_G \qquad N_B - M g = 0 \qquad N_B(b-a) - F_B h = 0$

$$\begin{pmatrix} N_B \\ F_B \\ a_{G2} \end{pmatrix} = \text{Find}(N_B, F_B, a_G) \qquad \begin{pmatrix} N_B \\ F_B \end{pmatrix} = \begin{pmatrix} 9565 \\ 6608 \end{pmatrix} \text{N} \qquad a_{G2} = 6.78\ \frac{\text{m}}{\text{s}^2}$$

Choose the critical case $a_G = \min(a_{G1}, a_{G2}) \qquad a_G = 6.78\ \frac{\text{m}}{\text{s}^2}$

Problem 17-26



The bottle of weight W rests on the check-out conveyor at a grocery store. If the coefficient of static friction is μ_s, determine the largest acceleration the conveyor can have without causing the bottle to slip or tip. The center of gravity is at G.

Given:

$W = 2\ \text{lb}$

$\mu_s = 0.2$

$b = 8 \text{ in}$

$c = 1.5 \text{ in}$

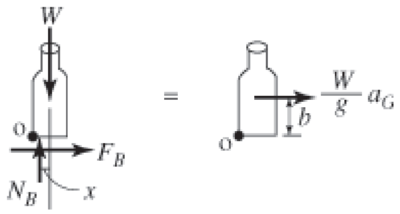
$g = 32.2 \frac{\text{ft}}{\text{s}^2}$

Solution: Assume that bottle tips before slipping

$x = c$

Guesses $a_G = 1 \frac{\text{ft}}{\text{s}^2}$ $F_B = 1 \text{ lb}$ $N_B = 1 \text{ lb}$ $F_{max} = 1 \text{ lb}$

Given $F_B = \left(\frac{W}{g}\right) a_G$ $N_B - W = 0$



$F_B b - N_B x = 0$ $F_{max} = \mu_s N_B$

$$\begin{pmatrix} a_{Gt} \\ F_B \\ N_B \\ F_{max} \end{pmatrix} = \text{Find}(a_G, F_B, N_B, F_{max}) \qquad \begin{pmatrix} F_B \\ N_B \\ F_{max} \end{pmatrix} = \begin{pmatrix} 0.375 \\ 2 \\ 0.4 \end{pmatrix} \text{lb} \qquad a_{Gt} = 6.037 \frac{\text{ft}}{\text{s}^2}$$

If $F_B = 0.375 \text{ lb} < F_{max} = 0.4 \text{ lb}$ then we have the correct answer.

If $F_B = 0.375 \text{ lb} > F_{max} = 0.4 \text{ lb}$ then we know that slipping occurs first. If this is the case,

$F_B = \mu_s N_B$

Given $F_B = \left(\frac{W}{g}\right) a_G$ $N_B - W = 0$ $F_B b - N_B x = 0$

$$\begin{pmatrix} a_{Gs} \\ N_B \\ x \end{pmatrix} = \text{Find}(a_G, N_B, x) \qquad N_B = 2 \text{ lb} \qquad x = 1.6 \text{ in} \qquad a_{Gs} = 6.44 \frac{\text{ft}}{\text{s}^2}$$

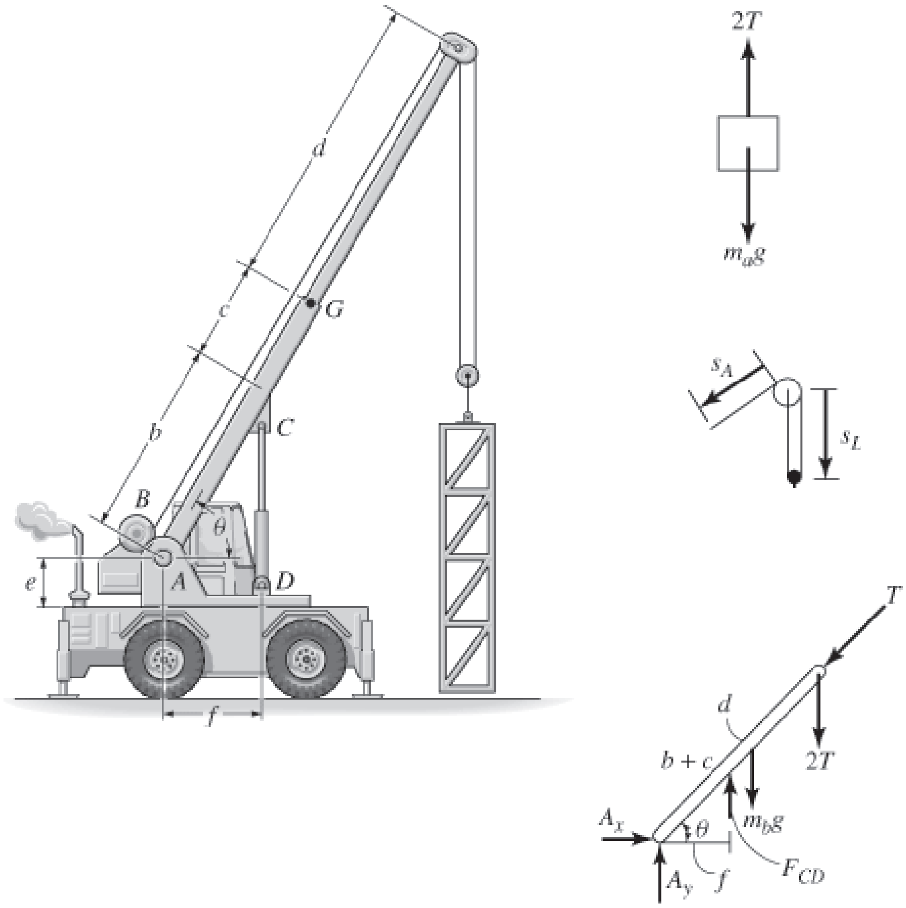
As a check, we should have $x = 1.6 \text{ in} < c = 1.5 \text{ in}$ if slipping occurs first

In either case, the answer is $a_G = \min(a_{Gs}, a_{Gt})$ $\boxed{a_G = 6.037 \frac{\text{ft}}{\text{s}^2}}$

Problem 17-27

The assembly has mass m_a and is hoisted using the boom and pulley system. If the winch at B

draws in the cable with acceleration a, determine the compressive force in the hydraulic cylinder needed to support the boom. The boom has mass m_b and mass center at G.



Units Used:

$\text{Mg} = 10^3 \text{ kg}$

$\text{kN} = 10^3 \text{ N}$

Given:

$m_a = 8 \text{ Mg}$

$m_b = 2 \text{ Mg}$

$a = 2 \frac{\text{m}}{\text{s}^2}$

$b = 4 \text{ m}$

$c = 2 \text{ m}$

$d = 6 \text{ m}$

$e = 1 \text{ m}$

$f = 2 \text{ m}$

$\theta = 60 \text{ deg}$

$g = 9.81 \frac{\text{m}}{\text{s}^2}$

Solution:

$$2T - m_a g = m_a \frac{a}{2} \qquad T = \frac{1}{2}\left(m_a g + m_a \frac{a}{2}\right) \qquad T = 43.24 \text{ kN}$$

$$\left[-2T(b + c + d) - m_b g(b + c) + F_{CD} b\right]\cos(\theta) = 0$$

$$F_{CD} = \frac{2T(b + c + d) + m_b g(b + c)}{b} \qquad F_{CD} = 289 \text{ kN}$$

***Problem 17-28**

The jet aircraft has total mass M and a center of mass at G. Initially at take-off the engines provide thrusts $2T$ and T'. Determine the acceleration of the plane and the normal reactions on the nose wheel and each of the *two* wing wheels located at B. Neglect the mass of the wheels and, due to low velocity, neglect any lift caused by the wings.

Units Used:

$\text{Mg} = 10^3 \text{ kg}$

$\text{kN} = 10^3 \text{ N}$

Given:

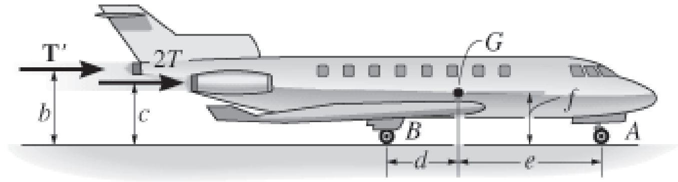
$b = 2.5 \text{ m}$ $\quad M = 22 \text{ Mg}$

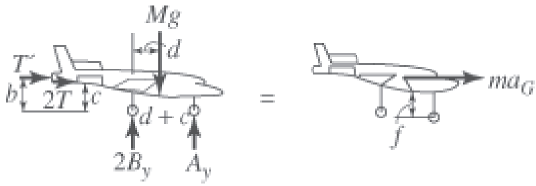
$c = 2.3 \text{ m}$ $\quad T = 2 \text{ kN}$

$d = 3 \text{ m}$ $\quad T' = 1.5 \text{ kN}$

$e = 6 \text{ m}$ $\quad g = 9.81 \dfrac{\text{m}}{\text{s}^2}$

$f = 1.2 \text{ m}$





Solution:

Guesses $\quad a_G = 1 \dfrac{\text{m}}{\text{s}^2} \qquad B_y = 1 \text{ kN} \qquad A_y = 1 \text{ kN}$

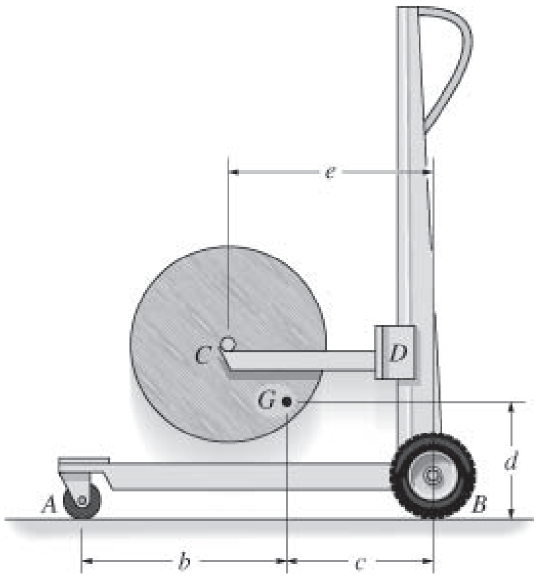
Given $\quad T' + 2T = M a_G \qquad 2B_y + A_y - M g = 0$

$$-T' b - 2T c - M g d + A_y(d + e) = -M a_G f$$

$$\begin{pmatrix} a_G \\ B_y \\ A_y \end{pmatrix} = \text{Find}(a_G, B_y, A_y) \qquad a_G = 0.250 \frac{\text{m}}{\text{s}^2} \qquad \begin{pmatrix} A_y \\ B_y \end{pmatrix} = \begin{pmatrix} 72.6 \\ 71.6 \end{pmatrix} \text{kN}$$

Problem 17-29

The lift truck has mass m_t and mass center at G. If it lifts the spool of mass m_s with acceleration a, determine the reactions of each of the four wheels on the ground. The loading is symmetric. Neglect the mass of the movable arm CD.



Given: $\quad a = 3 \dfrac{\text{m}}{\text{s}^2} \qquad d = 0.4 \text{ m}$

$b = 0.75 \text{ m} \qquad e = 0.7 \text{ m}$

$c = 0.5 \text{ m} \qquad g = 9.81 \dfrac{\text{m}}{\text{s}^2}$

$m_t = 70 \text{ kg} \qquad m_s = 120 \text{ kg}$

Solution:

Guesses $N_A = 1\ \text{N}$ $N_B = 1\ \text{N}$

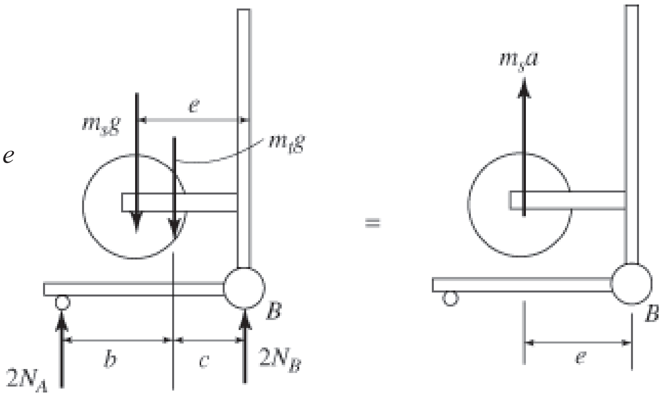
Given

$$2(N_A + N_B) - (m_t + m_s)g = m_s a$$

$$-2N_A(b + c) + m_s g e + m_t g c = -m_s a e$$

$$\begin{pmatrix} N_A \\ N_B \end{pmatrix} = \text{Find}(N_A, N_B)$$

$$\begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 568 \\ 544 \end{pmatrix} \text{N}$$



Problem 17-30

The lift truck has mass m_t and mass center at G. Determine the largest upward acceleration of the spool of mass m_s so that no reaction of the wheels on the ground exceeds F_{max}.

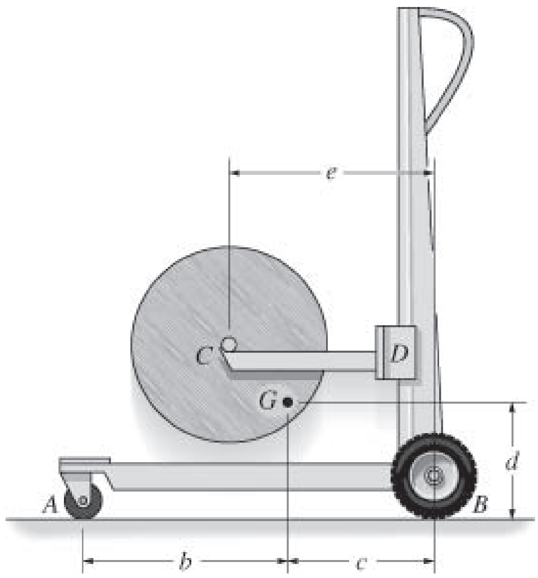
Given:

$m_t = 70\ \text{kg}$ $b = 0.75\ \text{m}$

$m_s = 120\ \text{kg}$ $c = 0.5\ \text{m}$

$F_{max} = 600\ \text{N}$ $d = 0.4\ \text{m}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$ $e = 0.7\ \text{m}$



Solution: Assume $N_A = F_{max}$

Guesses $a = 1\ \dfrac{\text{m}}{\text{s}^2}$ $N_B = 1\ \text{N}$

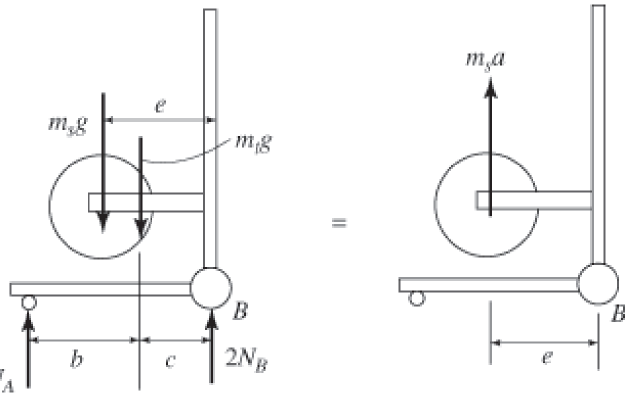
Given

$$2(N_A + N_B) - (m_t + m_s)g = m_s a$$

$$-2N_A(b + c) + m_s g e + m_t g c = -m_s a e$$

$$\begin{pmatrix} a \\ N_B \end{pmatrix} = \text{Find}(a, N_B)$$

$$\begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 600 \\ 569.529 \end{pmatrix} \text{N}$$



$$a = 3.96 \ \frac{\text{m}}{\text{s}^2}$$

Check: Since $N_B = 570$ N $<$ $F_{max} = 600$ N then our assumption is good.

Problem 17-31

The door has weight W and center of gravity at G. Determine how far the door moves in time t starting from rest, if a man pushes on it at C with a horizontal force F. Also, find the vertical reactions at the rollers A and B.

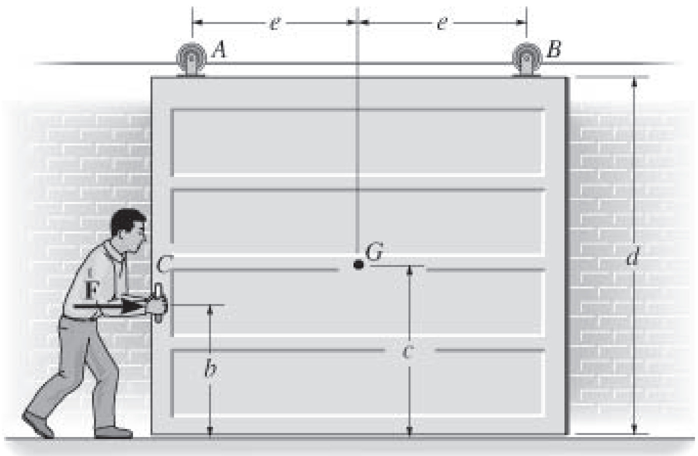
Given:

$W = 200$ lb $\quad c = 5$ ft

$t = 2$ s $\quad d = 12$ ft

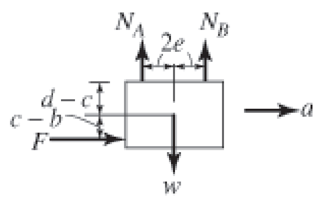
$F = 30$ lb $\quad e = 6$ ft

$b = 3$ ft $\quad g = 32.2 \ \frac{\text{ft}}{\text{s}^2}$



Solution:

Guesses $\quad a = 1 \ \frac{\text{ft}}{\text{s}^2} \quad N_A = 1$ lb $\quad N_B = 1$ lb



Given $\quad F = \left(\frac{W}{g}\right)a \qquad N_A + N_B - W = 0$

$$F(c - b) + N_B e - N_A e = 0$$

$$\begin{pmatrix} a \\ N_A \\ N_B \end{pmatrix} = \text{Find}(a, N_A, N_B) \qquad a = 4.83\,\frac{\text{ft}}{\text{s}^2} \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 105.0 \\ 95.0 \end{pmatrix} \text{lb}$$

$$d = \frac{1}{2} a t^2 \qquad d = 9.66\,\text{ft}$$

***Problem 17-32**

The door has weight W and center of gravity at G. Determine the constant force F that must be applied to the door to push it open a distance d to the right in time t, starting from rest. Also, find the vertical reactions at the rollers A and B.

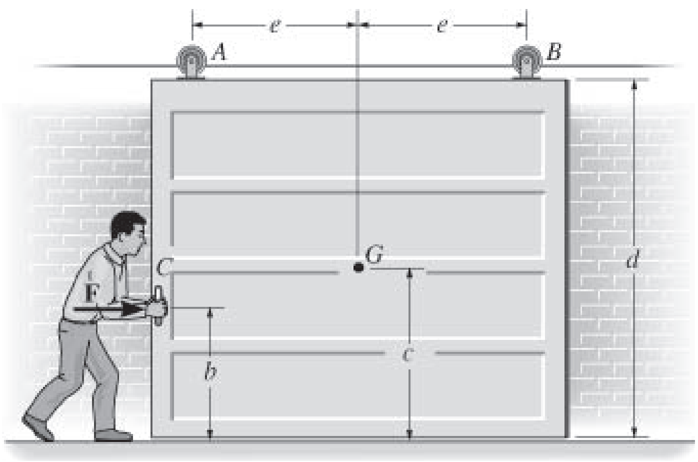
Given:

$W = 200\ \text{lb}$ $\quad c = 5\ \text{ft}$

$t = 5\ \text{s}$ $\quad d = 12\ \text{ft}$

$d = 12\ \text{ft}$ $\quad e = 6\ \text{ft}$

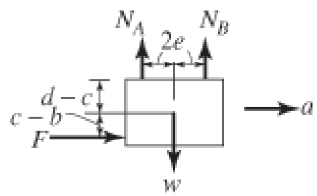
$b = 3\ \text{ft}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

$$a = 2\left(\frac{d}{t^2}\right) \qquad a = 0.96\,\frac{\text{ft}}{\text{s}^2}$$

Guesses $\quad F = 1\ \text{lb} \quad N_A = 1\ \text{lb} \quad N_B = 1\ \text{lb}$



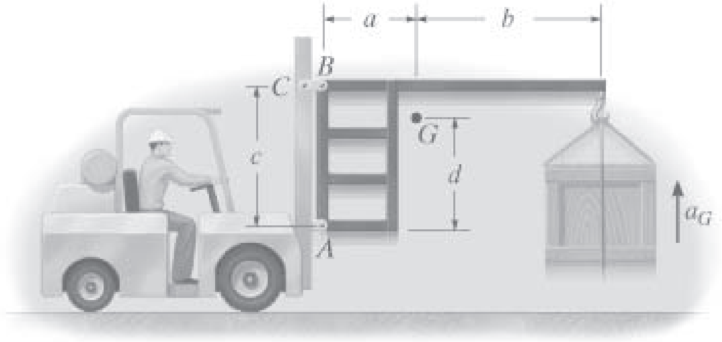
Given $\quad F = \left(\frac{W}{g}\right)a \qquad N_A + N_B - W = 0$

$$F(c - b) + N_B e - N_A e = 0$$

$$\begin{pmatrix} F \\ N_A \\ N_B \end{pmatrix} = \text{Find}(F, N_A, N_B) \qquad \begin{pmatrix} F \\ N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 5.96 \\ 100.99 \\ 99.01 \end{pmatrix} \text{lb}$$

Problem 17-33

The fork lift has a boom with mass M_1 and a mass center at G. If the vertical acceleration of the boom is a_G, determine the horizontal and vertical reactions at the pin A and on the short link BC when the load M_2 is lifted.



Units Used:

$\mathrm{Mg} = 10^3\ \mathrm{kg}$ $\quad$ $\mathrm{kN} = 10^3\ \mathrm{N}$

Given:

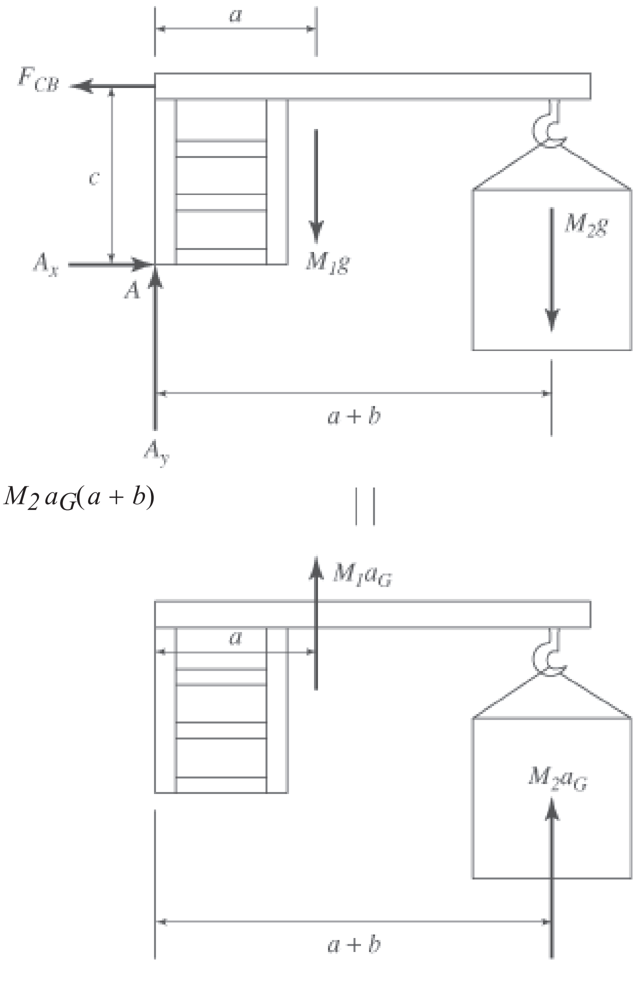
$M_1 = 800\ \mathrm{kg}$ $\quad$ $a = 1\ \mathrm{m}$

$M_2 = 1.25\ \mathrm{Mg}$ $\quad$ $b = 2\ \mathrm{m}$

$c = 1.5\ \mathrm{m}$

$a_G = 4\ \dfrac{\mathrm{m}}{\mathrm{s}^2}$ $\quad$ $d = 1.25\ \mathrm{m}$

Solution:



Guesses

$A_x = 1\ \mathrm{N}$ $\quad$ $F_{CB} = 1\ \mathrm{N}$

$A_y = 1\ \mathrm{N}$

Given

$$-F_{CB} + A_x = 0$$

$$A_y - (M_1 + M_2)g = (M_1 + M_2)a_G$$

$$F_{CB}\,c - M_1\,g\,a - M_2\,g(a+b) = M_1\,a_G\,a + M_2\,a_G(a+b)$$

$$\begin{pmatrix} A_x \\ A_y \\ F_{CB} \end{pmatrix} = \mathrm{Find}(A_x, A_y, F_{CB})$$

$$\begin{pmatrix} A_x \\ A_y \\ F_{CB} \end{pmatrix} = \begin{pmatrix} 41.9 \\ 28.3 \\ 41.9 \end{pmatrix} \mathrm{kN}$$

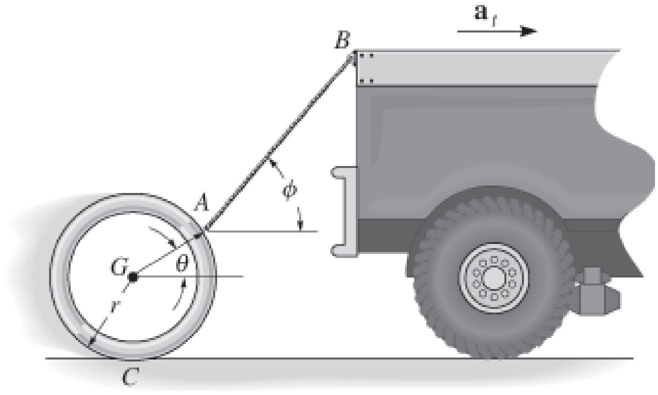
Problem 17-34

The pipe has mass M and is being towed behind the truck. If the acceleration of the truck is a_t, determine the angle θ and the tension in the cable. The coefficient of kinetic friction between the pipe and the ground is μ_k.

Units Used:

$$\text{kN} = 10^3\ \text{N}$$

Given:



$M = 800\ \text{kg}$ $\quad r = 0.4\ \text{m}$

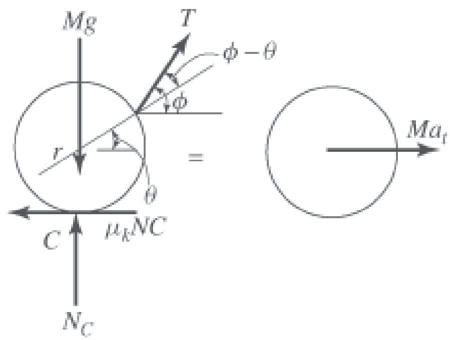
$a_t = 0.5\ \dfrac{\text{m}}{\text{s}^2}$ $\quad \phi = 45\ \text{deg}$

$\mu_k = 0.1$ $\quad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Solution:

Guesses $\quad \theta = 10\ \text{deg} \quad N_C = 1\ \text{N} \quad T = 1\ \text{N}$

Given



$$T\cos(\phi) - \mu_k N_C = M a_t$$

$$T\sin(\phi) - M g + N_C = 0$$

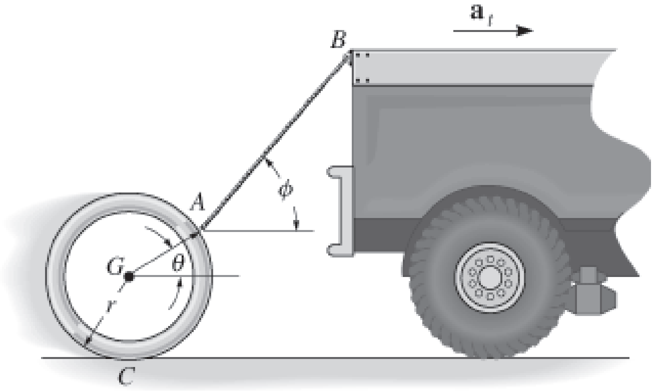
$$T\sin(\phi - \theta) r - \mu_k N_C r = 0$$

$$\begin{pmatrix} \theta \\ N_C \\ T \end{pmatrix} = \text{Find}(\theta, N_C, T) \qquad N_C = 6.771\ \text{kN} \qquad T = 1.523\ \text{kN} \qquad \theta = 18.608\ \text{deg}$$

Problem 17-35

The pipe has mass M and is being towed behind a truck. Determine the acceleration of the truck and the tension in the cable. The coefficient of kinetic friction between the pipe and the ground is μ_k.

Units Used: $\text{kN} = 10^3 \text{ N}$



Given:

$M = 800 \text{ kg}$ $r = 0.4 \text{ m}$

$\theta = 30 \text{ deg}$ $\phi = 45 \text{ deg}$

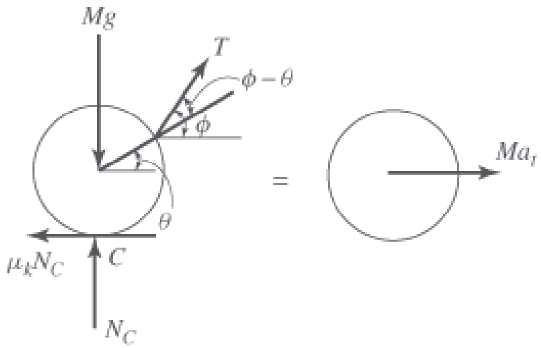
$\mu_k = 0.1$ $g = 9.81 \frac{\text{m}}{\text{s}^2}$

Solution:

Guesses $a_t = 1 \frac{\text{m}}{\text{s}^2}$

$N_C = 1 \text{ N}$

$T = 1 \text{ N}$



Given

$$T\cos(\phi) - \mu_k N_C = M a_t$$

$$T\sin(\phi) - M g + N_C = 0$$

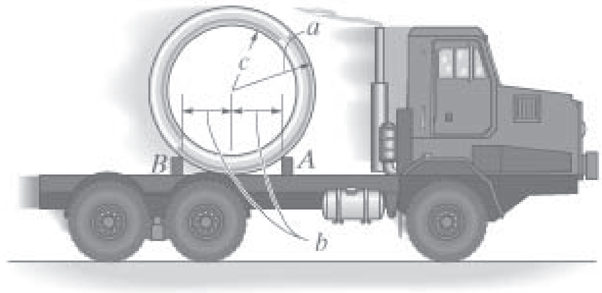
$$T\sin(\phi - \theta)r - \mu_k N_C r = 0$$

$$\begin{pmatrix} a_t \\ N_C \\ T \end{pmatrix} = \text{Find}(a_t, N_C, T) \qquad N_C = 6.164 \text{ kN} \qquad T = 2.382 \text{ kN} \qquad a_t = 1.335 \frac{\text{m}}{\text{s}^2}$$

***Problem 17-36**

The pipe has a mass M and is held in place on the truck bed using the two boards A and B. Determine the acceleration of the truck so that the pipe begins to lose contact at A and the bed of the truck and starts to pivot about B. Assume board B will not slip on the bed of the truck, and the pipe is smooth. Also, what force does board B exert on the pipe during the acceleration?

Units Used: $\text{kN} = 10^3\ \text{N}$



Given:

$M = 460\ \text{kg}$

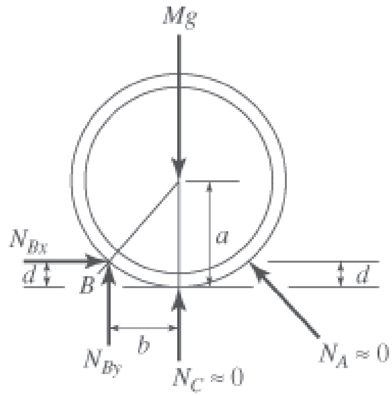
$a = 0.5\ \text{m}$

$b = 0.3\ \text{m}$

$c = 0.4\ \text{m}$

$d = 0.1\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution: $\theta = \text{asin}\left(\frac{b}{a}\right)$

Guesses

$N_{Bx} = 1\ \text{N}$ $N_{By} = 1\ \text{N}$ $a_t = 1\ \frac{\text{m}}{\text{s}^2}$

Given

$N_{Bx} = M a_t$ $N_{By} - M g = 0$ $N_{Bx}(a)\cos(\theta) - N_{By}\, b = 0$

$$\begin{pmatrix} N_{Bx} \\ N_{By} \\ a_t \end{pmatrix} = \text{Find}(N_{Bx}, N_{By}, a_t) \qquad \begin{pmatrix} N_{Bx} \\ N_{By} \end{pmatrix} = \begin{pmatrix} 3.384 \\ 4.513 \end{pmatrix}\text{kN} \qquad a_t = 7.36\ \frac{\text{m}}{\text{s}^2}$$

$$\left|\begin{pmatrix} N_{Bx} \\ N_{By} \end{pmatrix}\right| = 5.64\ \text{kN}$$

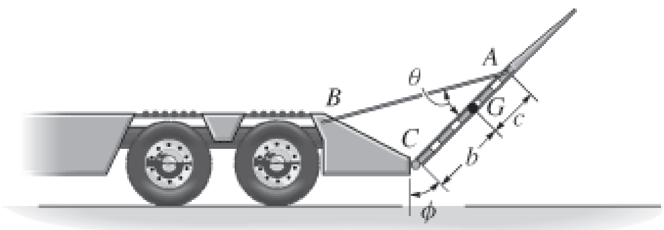
Problem 17-37

The drop gate at the end of the trailer has mass M and mass center at G. If it is supported by the cable AB and hinge at C, determine the tension in the cable when the truck begins to accelerate at rate a. Also, what are the horizontal and vertical components of reaction at the hinge C?

Given: $\text{kN} = 10^3\ \text{N}$

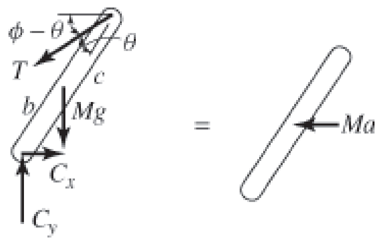
$M = 1.25 \times 10^3\ \text{kg}$

$a = 5\ \frac{\text{m}}{\text{s}^2}$ $\theta = 30\ \text{deg}$



$b = 1.5\ \text{m} \qquad \phi = 45\ \text{deg}$

$c = 1\ \text{m} \qquad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$



Solution:

Guesses $\quad T = 1\ \text{N} \quad C_x = 1\ \text{N} \quad C_y = 1\ \text{N}$

Given $\quad -T\cos(\phi - \theta) + C_x = -M\,a$

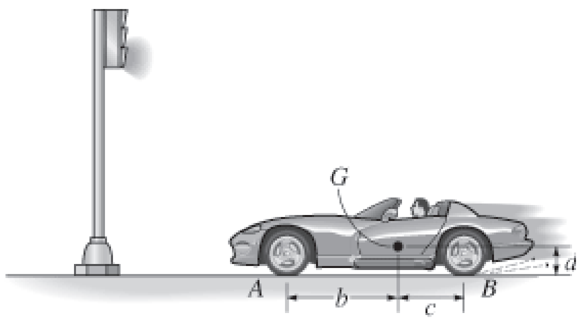
$$-T\sin(\phi - \theta) - M\,g + C_y = 0$$

$$T\sin(\theta)(b + c) - M\,g\,b\cos(\phi) = M\,a\,b\sin(\phi)$$

$$\begin{pmatrix} T \\ C_x \\ C_y \end{pmatrix} = \text{Find}(T, C_x, C_y) \qquad \begin{pmatrix} T \\ C_x \\ C_y \end{pmatrix} = \begin{pmatrix} 15.708 \\ 8.923 \\ 16.328 \end{pmatrix} \text{kN}$$

Problem 17-38

The sports car has mass M and a center of mass at G. Determine the shortest time it takes for it to reach speed v, starting from rest, if the engine only drives the rear wheels, whereas the front wheels are free rolling. The coefficient of static friction between the wheels and the road is μ_s. Neglect the mass of the wheels for the calculation. If driving power could be supplied to all four wheels, what would be the shortest time for the car to reach a speed of v?



Given:

$M = 1.5 \times 10^3\ \text{kg}$

$v = 80\ \dfrac{\text{km}}{\text{hr}}$

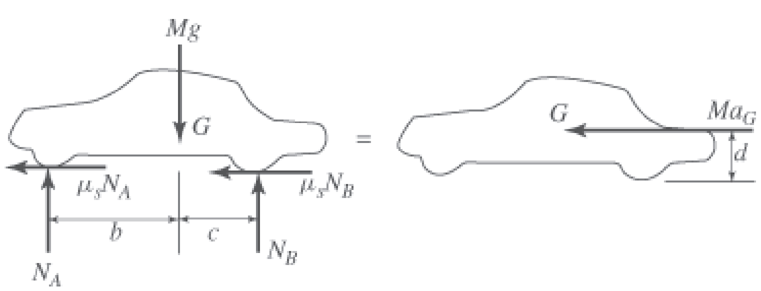
$\mu_s = 0.2$

$b = 1.25\ \text{m}$

$c = 0.75\ \text{m}$

$d = 0.35\ \text{m}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$



Solution:

(a) Rear wheel drive only Guesses $N_A = 1\ \text{N}$ $N_B = 1\ \text{N}$ $a_G = 1\ \dfrac{\text{m}}{\text{s}^2}$

Given $N_A + N_B - Mg = 0$ $\mu_s N_B = M a_G$

$Mgc - N_A(b + c) = M a_G d$

$$\begin{pmatrix} N_A \\ N_B \\ a_G \end{pmatrix} = \text{Find}(N_A, N_B, a_G) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 5.185 \times 10^3 \\ 9.53 \times 10^3 \end{pmatrix} \text{N} \qquad a_G = 1.271\ \frac{\text{m}}{\text{s}^2}$$

$$t_{rw} = \frac{v}{a_G} \qquad \boxed{t_{rw} = 17.488\ \text{s}}$$

(b) Four wheel drive Guesses $N_A = 1\ \text{N}$ $N_B = 1\ \text{N}$ $a_G = 1\ \dfrac{\text{m}}{\text{s}^2}$

Given $N_A + N_B - Mg = 0$ $\mu_s N_B + \mu_s N_A = M a_G$

$Mgc - N_A(b + c) = M a_G d$

$$\begin{pmatrix} N_A \\ N_B \\ a_G \end{pmatrix} = \text{Find}(N_A, N_B, a_G) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 5.003 \times 10^3 \\ 9.712 \times 10^3 \end{pmatrix} \text{N} \qquad a_G = 1.962\ \frac{\text{m}}{\text{s}^2}$$

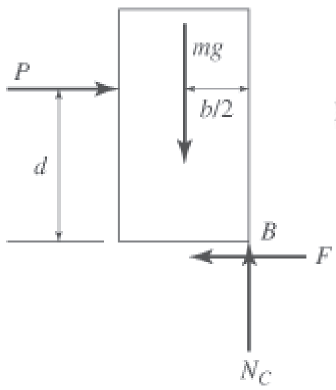
$$t_{rw} = \frac{v}{a_G} \qquad \boxed{t_{rw} = 11.326\ \text{s}}$$

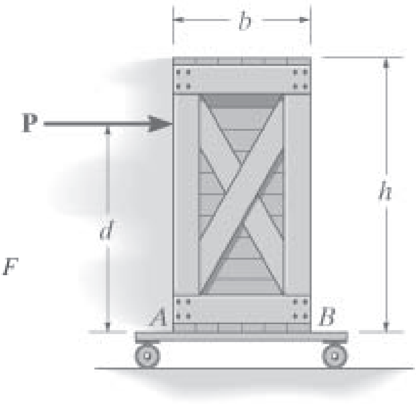
Problem 17-39

The crate of mass m is supported on a cart of negligible mass. Determine the maximum force P that can be applied a distance d from the cart bottom without causing the crate to tip on the cart.

Solution:

Require N_c to act at corner B for tipping.





$$P\,d - m\,g\left(\frac{b}{2}\right) = m\,a_G\left(\frac{h}{2}\right)$$

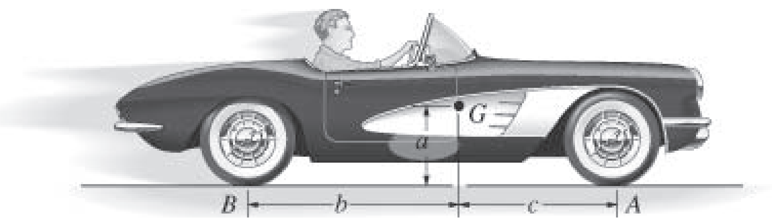
$$P = m\,a_G$$

$$P\,d - m\mathbf{g}\left(\frac{b}{2}\right) = P\left(\frac{h}{2}\right)$$

$$\boxed{P_{max} = \frac{m\,g\,b}{2\left(d - \frac{h}{2}\right)}}$$

***Problem 17-40**

The car accelerates uniformly from rest to speed v in time t. If it has weight W and a center of gravity at G, determine the normal reaction of *each wheel* on the pavement during the motion. Power is developed at the front wheels, whereas the rear wheels are free to roll. Neglect the mass of the wheels and take the coefficients of static and kinetic friction to be μ_s and μ_k respectively.



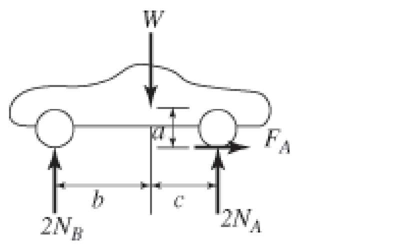
Given: $a = 2.5\ \text{ft}$

$v = 88\ \frac{\text{ft}}{\text{s}}$ $\mu_k = 0.2$

$t = 15\ \text{s}$ $b = 4\ \text{ft}$

$W = 3800\ \text{lb}$ $c = 3\ \text{ft}$

$\mu_s = 0.4$ $g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution: Assume no slipping $a_G = \frac{v}{t}$ $a_G = 5.867\ \frac{\text{ft}}{\text{s}^2}$

Guesses $N_B = 1\ \text{lb}$ $N_A = 1\ \text{lb}$ $F_A = 1\ \text{lb}$

Given $2N_B + 2N_A - W = 0$

$$2F_A = \left(\frac{W}{g}\right)a_G$$

$$-2N_B(b + c) + W\,c = \left(\frac{-W}{g}\right)a_G\,a$$

$$\begin{pmatrix} N_A \\ N_B \\ F_A \end{pmatrix} = \text{Find}(N_A, N_B, F_A) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 962 \\ 938 \end{pmatrix} \text{lb} \qquad F_A = 346\,\text{lb}$$

$$F_{max} = \mu_s N_A \qquad F_{max} = 385\,\text{lb}$$

Check: Our no-slip assumption is true if $F_A = 346\,\text{lb} < F_{max} = 385\,\text{lb}$

Problem 17-41

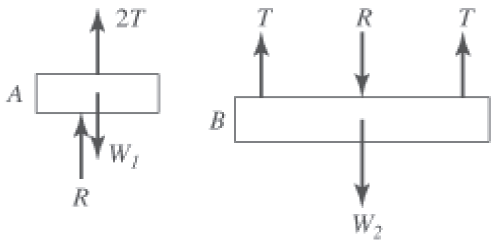
Block A has weight W_1 and the platform has weight W_2. Determine the normal force exerted by block A on B. Neglect the weight of the pulleys and bars of the triangular frame.

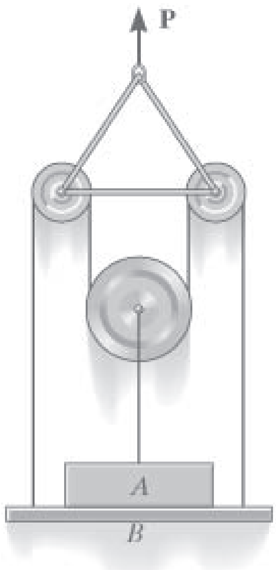
Given:

$W_1 = 50$ lb

$W_2 = 10$ lb

$P = 100$ lb

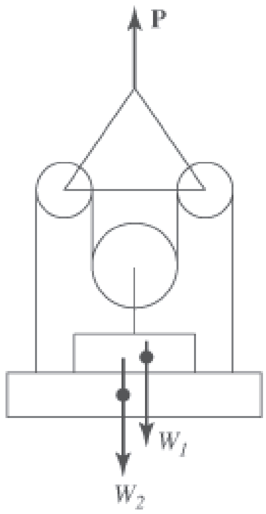




Solution:

$$P - (W_1 + W_2) = \left(\frac{W_1 + W_2}{g}\right) a_G$$

$$a_G = g\left(\frac{P - 1\,W_1 - 1\,W_2}{W_1 + W_2}\right) \qquad a_G = 21.47\,\frac{\text{ft}}{\text{s}^2}$$



Guesses $\qquad R = 25$ lb $\qquad T = 22$ lb

Given

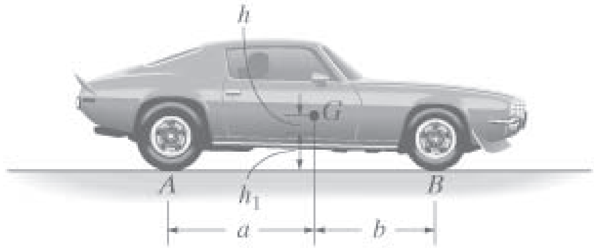
$$2T + R - W_1 = \left(\frac{W_1}{g}\right) a_G$$

$$2T - R - W_2 = \left(\frac{W_2}{g}\right) a_G$$

$$\begin{pmatrix} R \\ T \end{pmatrix} = \text{Find}(R, T) \qquad R = 33.3\,\text{lb} \qquad T = 25\,\text{lb}$$

Problem 17-42

The car of mass M shown has been "raked" by increasing the height of its center of mass to h This was done by raising the springs on the rear axle. If the coefficient of kinetic friction between the rear wheels and the ground is μ_k, show that the car can accelerate slightly faster than its counterpart for which $h = 0$. Neglect the mass of the wheels and driver and assume the front wheels at B are free to roll while the rear wheels slip.



Units Used:

$$\text{Mg} = 10^3\ \text{kg} \quad \text{kN} = 10^3\ \text{N}$$

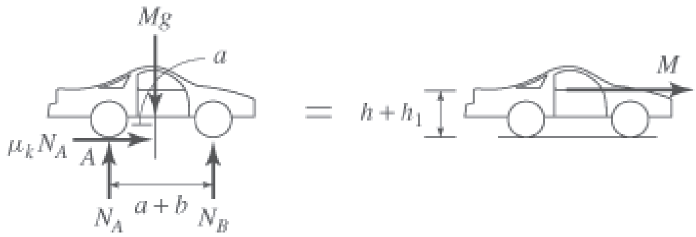
Given:

$$M = 1.6\ \text{Mg} \quad a = 1.6\ \text{m}$$

$$\mu_k = 0.3 \quad b = 1.3\ \text{m}$$

$$g = 9.81\ \frac{\text{m}}{\text{s}^2} \quad h = 0.2\ \text{m}$$

$$h_1 = 0.4\ \text{m}$$



Solution:

In the raised position

Guesses $\quad a_G = 1\ \frac{\text{m}}{\text{s}^2} \quad N_A = 4\ \text{N} \quad N_B = 5\ \text{N}$

Given $\quad \mu_k N_A = M a_G$

$$N_A + N_B - M g = 0$$

$$-M g a + N_B(a + b) = -M a_G(h + h_1)$$

$$\begin{pmatrix} a_{Gr} \\ N_A \\ N_B \end{pmatrix} = \text{Find}(a_G, N_A, N_B) \qquad a_{Gr} = 1.41\ \frac{\text{m}}{\text{s}^2}$$

In the lower (regular) position

Given

$$\mu_k N_A = M a_G \qquad N_A + N_B - M g = 0 \qquad -M g a + N_B(a + b) = -M a_G h_1$$

$$\begin{pmatrix} a_{Gl} \\ N_A \\ N_B \end{pmatrix} = \text{Find}(a_G, N_A, N_B) \qquad a_{Gl} = 1.38\ \frac{\text{m}}{\text{s}^2}$$

Thus the advantage in the raised position is $\quad a_{Gr} - a_{Gl} = 0.03\ \frac{\text{m}}{\text{s}^2}$

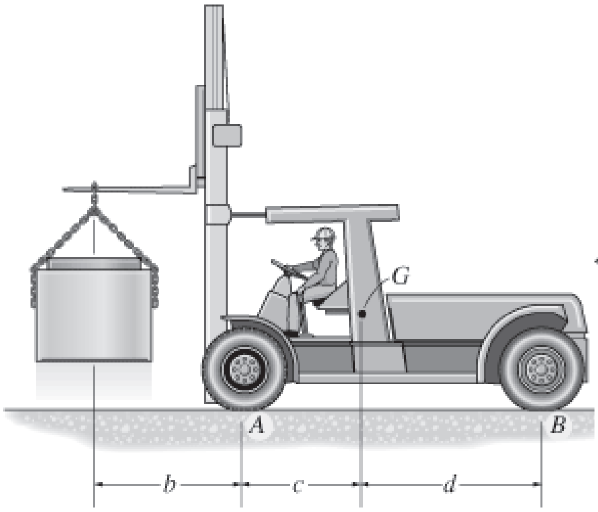
Problem 17-43

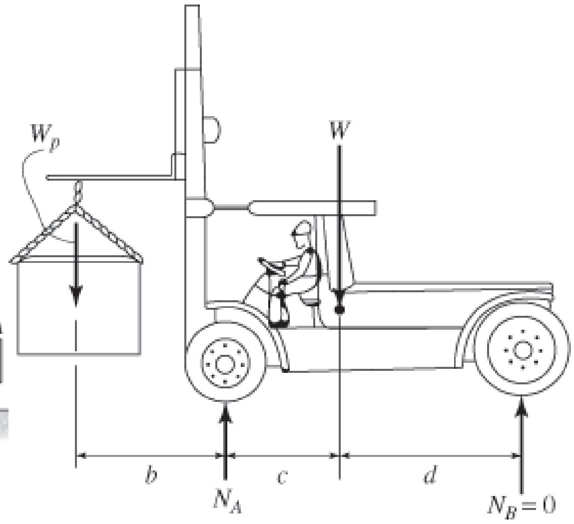
The forklift and operator have combined weight W and center of mass at G. If the forklift is used to lift the concrete pipe of weight W_p determine the maximum vertical acceleration it can give to the pipe so that it does not tip forward on its front wheels.

Given:

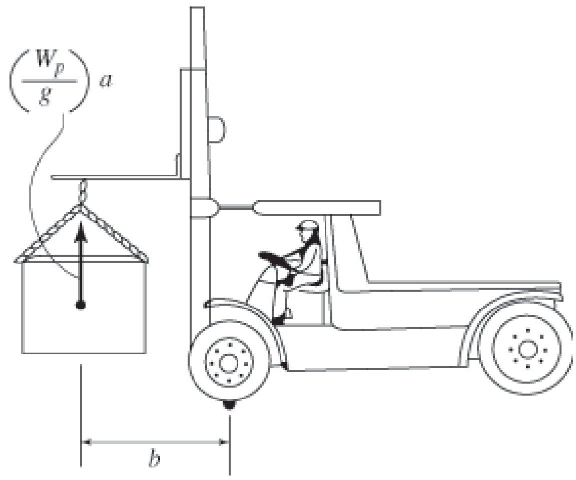
$W = 10000 \text{ lb}$ $\quad b = 5 \text{ ft}$ $\quad d = 6 \text{ ft}$

$W_p = 2000 \text{ lb}$ $\quad c = 4 \text{ ft}$ $\quad g = 32.2 \dfrac{\text{ft}}{\text{s}^2}$





Solution:



It is required that $N_B = 0$

$$W_p b - W c = -\frac{W_p}{g} a b$$

$$a = \left(\frac{W c - W_p b}{W_p}\right)\frac{g}{b}$$

$$a = 96.6 \frac{\text{ft}}{\text{s}^2}$$

***Problem 17-44**

The forklift and operator have combined weight W and center of mass at G. If the forklift is used to lift the concrete pipe of weight W_p determine the normal reactions on each of its four wheels if the pipe is given upward acceleration a.

Units Used:

$$\text{kip} = 10^3 \text{ lb}$$

Given:

$$W_p = 2000 \text{ lb}$$

$$W = 10000 \text{ lb}$$

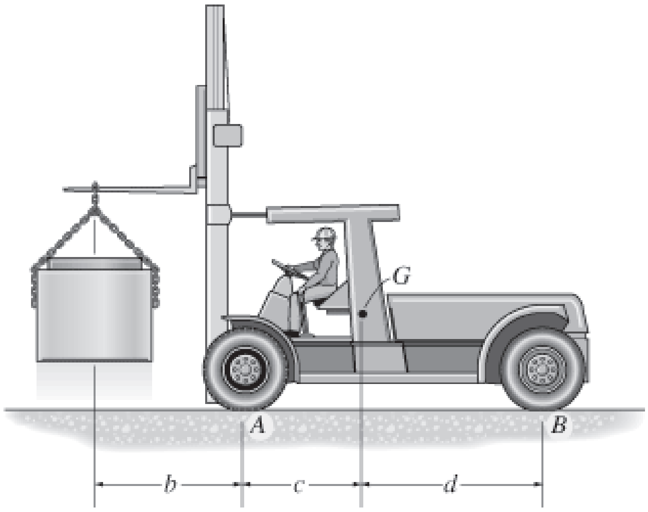
$$a = 4 \frac{\text{ft}}{\text{s}^2}$$

$$b = 5 \text{ ft}$$

$$c = 4 \text{ ft}$$

$$d = 6 \text{ ft}$$

$$g = 9.81 \frac{\text{m}}{\text{s}^2}$$



Solution:

Guesses

$$N_A = 1 \text{ lb}$$

$$N_B = 1 \text{ lb}$$

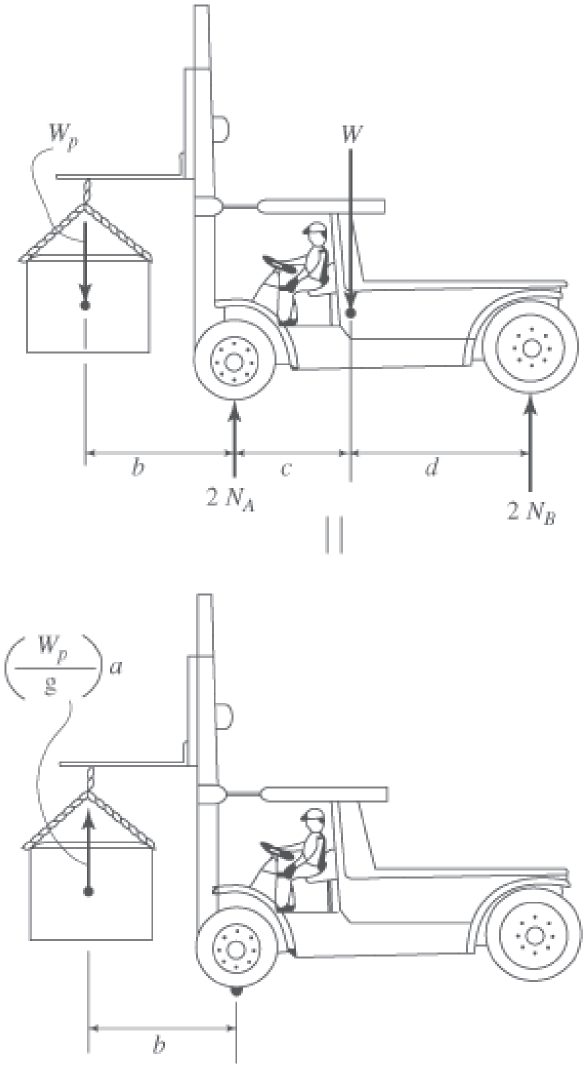
Given

$$2N_A + 2N_B - W - W_p = \left(\frac{W_p}{g}\right)a$$

$$2N_A b - W(b + c) + 2N_B(b + c + d) = 0$$

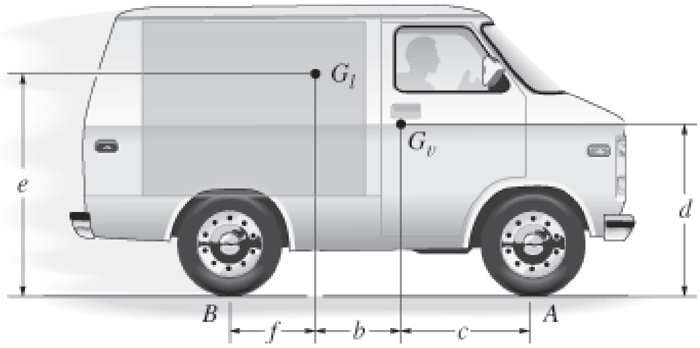
$$\begin{pmatrix} N_A \\ N_B \end{pmatrix} = \text{Find}(N_A, N_B)$$

$$\begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 4.686 \\ 1.438 \end{pmatrix} \text{kip}$$



Problem 17-45

The van has weight W_v and center of gravity at G_v. It carries fixed load W_l which has center of gravity at G_l. If the van is traveling at speed v, determine the distance it skids before stopping. The brakes cause *all* the wheels to lock or skid. The coefficient of kinetic friction between the wheels and the pavement is μ_k. Compare this distance with that of the van being empty. Neglect the mass of the wheels.



Given:

$W_v = 4500\ \text{lb}$ $\quad b = 2\ \text{ft}$

$W_l = 800\ \text{lb}$ $\quad c = 3\ \text{ft}$

$v = 40\ \frac{\text{ft}}{\text{s}}$ $\quad d = 4\ \text{ft}$

$\mu_k = 0.3$ $\quad e = 6\ \text{ft}$

$f = 2\ \text{ft}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution: Loaded

Guesses $N_A = 1\ \text{lb}$ $\quad N_B = 1\ \text{lb}$ $\quad a = 1\ \frac{\text{ft}}{\text{s}^2}$

Given

$$N_A + N_B - W_v - W_l = 0$$

$$\mu_k(N_A + N_B) = \left(\frac{W_v + W_l}{g}\right)a$$

$$-N_B(f + b + c) + W_l(b + c) + W_v c = \left(\frac{W_l}{g}\right)a\,e + \left(\frac{W_v}{g}\right)a\,d$$

$$\begin{pmatrix} N_A \\ N_B \\ a \end{pmatrix} = \text{Find}(N_A, N_B, a) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 3777 \\ 1523 \end{pmatrix}\text{lb}$$

$$a = 9.66\ \frac{\text{ft}}{\text{s}^2} \qquad d_l = \frac{v^2}{2a} \qquad d_l = 82.816\ \text{ft}$$

Unloaded $\quad W_l = 0\ \text{lb}$

Guesses $N_A = 1\ \text{lb}$ $\quad N_B = 1\ \text{lb}$ $\quad a = 1\ \frac{\text{ft}}{\text{s}^2}$

Given $\quad N_A + N_B - W_v - W_l = 0$

$$\mu_k(N_A + N_B) = \left(\frac{W_v + W_l}{g}\right)a$$

$$-N_B(f + b + c) + W_l(b + c) + W_v c = \left(\frac{W_l}{g}\right)a\,e + \left(\frac{W_v}{g}\right)a\,d$$

$$\begin{pmatrix} N_A \\ N_B \\ a \end{pmatrix} = \text{Find}(N_A, N_B, a) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 3343 \\ 1157 \end{pmatrix}\text{lb}$$

$$a = 9.66\,\frac{\text{ft}}{\text{s}^2} \qquad d_{ul} = \frac{v^2}{2a}$$

$$d_{ul} = 82.816\,\text{ft}$$

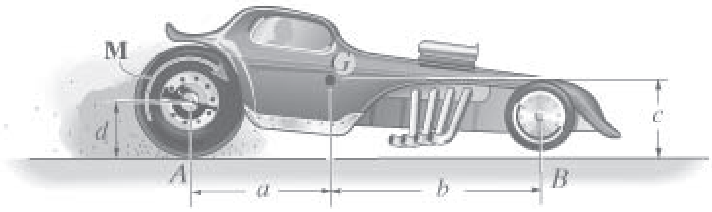
The distance is the same in both cases although the forces on the tires are different.

Problem 17-46

The "muscle car" is designed to do a "wheeley", i.e., to be able to lift its front wheels off the ground in the manner shown when it accelerates. If the car of mass M_1 has a center of mass at G, determine the minimum torque that must be developed at both rear wheels in order to do this. Also, what is the smallest necessary coefficient of static friction assuming the thick-walled rear wheels do not slip on the pavement? Neglect the mass of the wheels.

Units Used:

$\text{Mg} = 10^3\ \text{kg}$



$\text{kN} = 10^3\ \text{N}$

Given:

$M_1 = 1.35\ \text{Mg}$

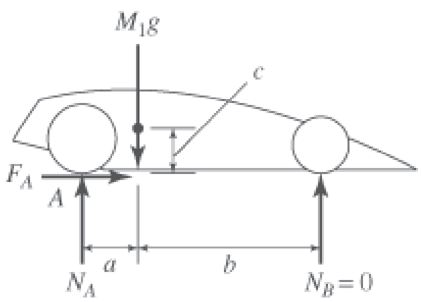
$a = 1.10\ \text{m}$

$b = 1.76\ \text{m}$

$c = 0.67\ \text{m}$

$d = 0.31\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution:

Guesses $\quad a_G = 1\ \frac{\text{m}}{\text{s}^2} \quad F_A = 1\ \text{N} \quad N_A = 1\ \text{N} \quad M = 1\ \text{N m} \quad \mu_s = 0.1$

Given

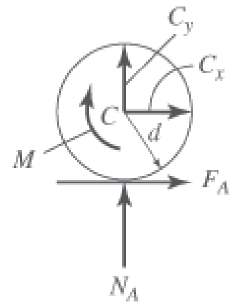
$F_A = M_1 a_G$

$N_A - M_1 g = 0$

$M_1 g a = M_1 a_G c$

$-M + F_A d = 0$

$F_A = \mu_s N_A$



$$\begin{pmatrix} a_G \\ F_A \\ N_A \\ M \\ \mu_s \end{pmatrix} = \text{Find}(a_G, F_A, N_A, M, \mu_s) \qquad \begin{pmatrix} F_A \\ N_A \end{pmatrix} = \begin{pmatrix} 21.7 \\ 13.2 \end{pmatrix} \text{kN} \qquad M = 6.74\ \text{kN}\cdot\text{m}$$

$$a_G = 16.11\ \frac{\text{m}}{\text{s}^2} \qquad \mu_s = 1.642$$

Problem 17-47

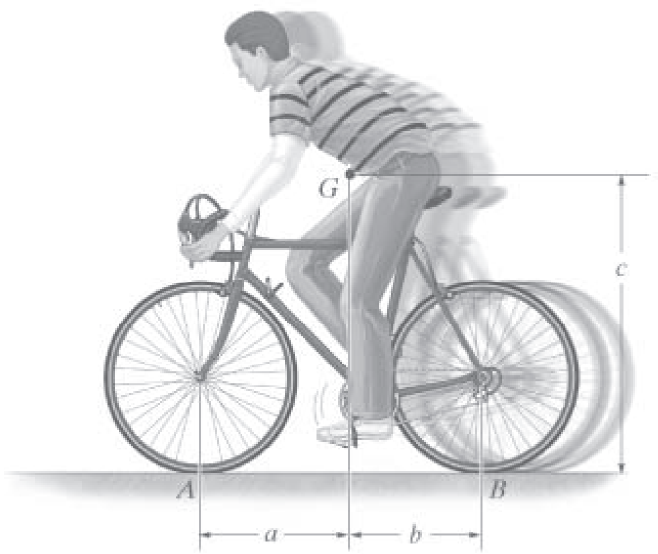
The bicycle and rider have a mass M with center of mass located at G. If the coefficient of kinetic friction at the rear tire is μ_B, determine the normal reactions at the tires A and B, and the deceleration of the rider, when the rear wheel locks for braking. What is the normal reaction at the rear wheel when the bicycle is traveling at constant velocity and the brakes are not applied? Neglect the mass of the wheels.

Given:

$M = 80\ \text{kg} \qquad \mu_B = 0.8$

$a = 0.55\ \text{m} \qquad b = 0.4\ \text{m}$

$c = 1.2\ \text{m}$



Solution: Deceleration:

Guesses $\quad a_G = 1\ \frac{\text{m}}{\text{s}^2}$

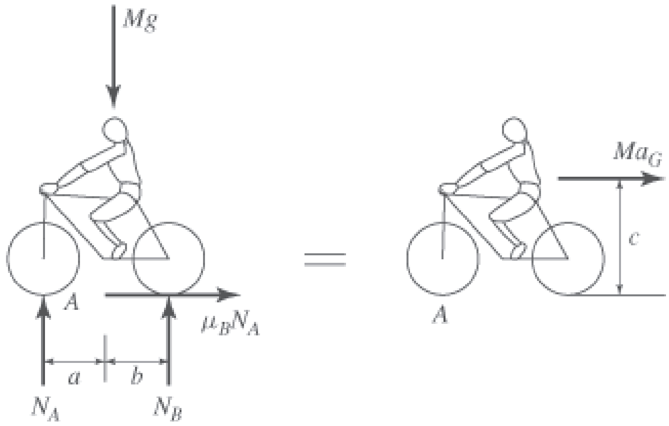
$N_B = 1\ \text{N} \qquad N_A = 1\ \text{N}$

Given $\quad \mu_B N_B = M a_G \qquad N_A + N_B - M g = 0$

$-N_B(a + b) + M g a = M a_G c$

$$\begin{pmatrix} a_G \\ N_B \\ N_A \end{pmatrix} = \text{Find}(a_G, N_B, N_A) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 559 \\ 226 \end{pmatrix} \text{N} \qquad a_G = 2.26\ \frac{\text{m}}{\text{s}^2}$$

Equilibrium



Given $N_A + N_B - Mg = 0$

$$-N_B(a+b) + Mga = 0$$

$$\begin{pmatrix} N_A \\ N_B \end{pmatrix} = \text{Find}(N_A, N_B)$$

$$\begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 330 \\ 454 \end{pmatrix} \text{N}$$

***Problem 17-48**

The bicycle and rider have a mass M with center of mass located at G. Determine the minimum coefficient of kinetic friction between the road and the wheels so that the rear wheel B starts to lift off the ground when the rider applies the brakes to the front wheel. Neglect the mass of the wheels.

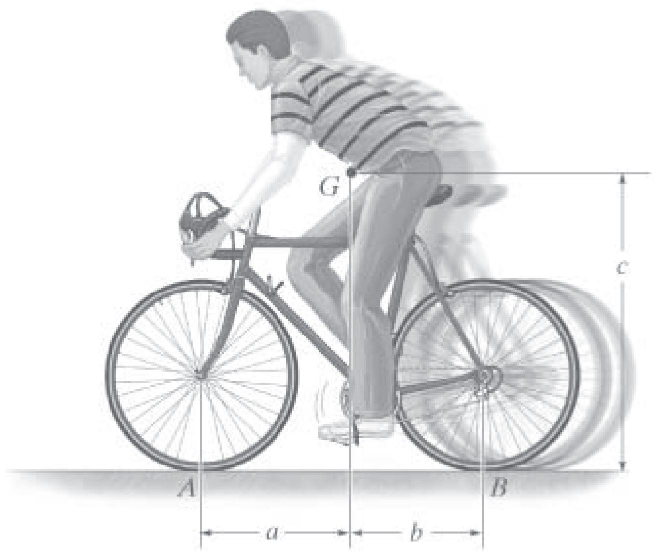
Given:

$M = 80$ kg

$a = 0.55$ m

$b = 0.4$ m

$c = 1.2$ m



Solution: $N_B = 0$

Guesses $a_G = 1\ \frac{\text{m}}{\text{s}^2}$

$\mu_k = 0.1$

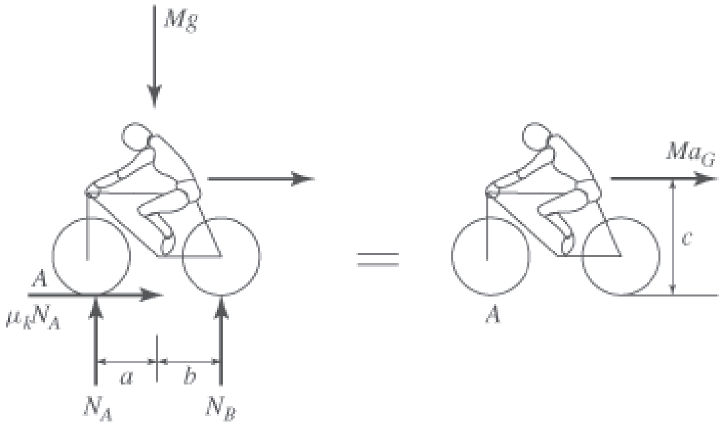
$N_A = 1$ N

Given $\mu_k N_A = M a_G$

$$N_A - Mg = 0$$

$$Mga = Ma_G c$$

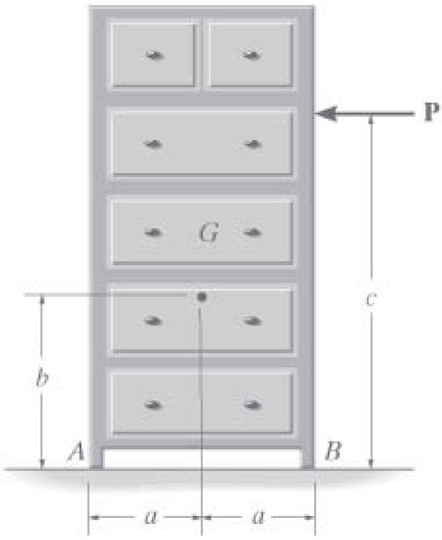
$$\begin{pmatrix} a_G \\ \mu_k \\ N_A \end{pmatrix} = \text{Find}(a_G, \mu_k, N_A)$$



$N_A = 785\ \text{N}$

$a_G = 4.50\ \dfrac{\text{m}}{\text{s}^2}$ $\quad$ $\mu_k = 0.458$

Problem 17-49



The dresser has a weight W and is pushed along the floor. If the coefficient of static friction at A and B is μ_s and the coefficient of kinetic friction is μ_k, determine the smallest horizontal force P needed to cause motion. If this force is increased slightly, determine the acceleration of the dresser. Also, what are the normal reactions at A and B when it begins to move?

Given:

$W = 80\ \text{lb}$

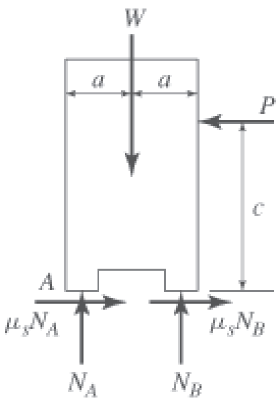
$\mu_s = 0.3$

$\mu_k = 0.2$

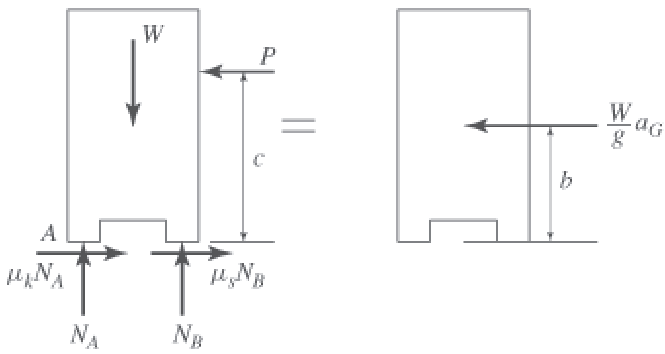
$a = 1.5\ \text{ft}$

$b = 2.5\ \text{ft}$

$c = 4\text{ft}$



Solution: Impending Motion



Guesses $\quad P = 1\ \text{lb}$

$N_A = 1\ \text{lb} \quad N_B = 1\ \text{lb}$

Given $\quad N_A + N_B - W = 0$

$$\mu_s N_A + \mu_s N_B - P = 0$$

$$P c + W a - N_A(2a) = 0$$

$$\begin{pmatrix} P \\ N_A \\ N_B \end{pmatrix} = \text{Find}(P, N_A, N_B) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 72 \\ 8 \end{pmatrix} \text{lb} \qquad P = 24\,\text{lb}$$

Motion Guesses $a_G = 1\ \frac{\text{ft}}{\text{s}^2}$

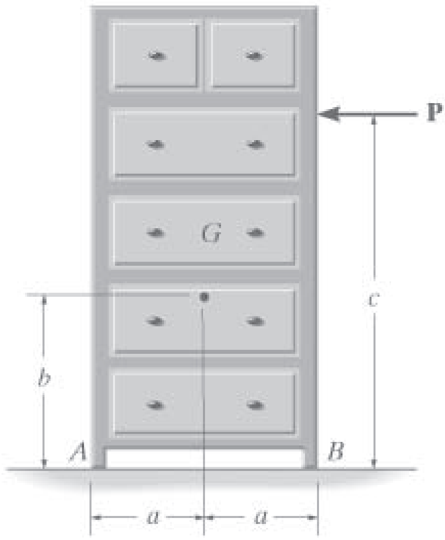
Given $N_A + N_B - W = 0 \qquad \mu_k N_A + \mu_k N_B - P = \left(\frac{-W}{g}\right) a_G$

$$P(c - b) + \mu_k(N_A + N_B)b + N_B a - N_A a = 0$$

$$\begin{pmatrix} N_A \\ N_B \\ a_G \end{pmatrix} = \text{Find}(N_A, N_B, a_G) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 65.3 \\ 14.7 \end{pmatrix} \text{lb} \qquad a_G = 3.22\ \frac{\text{ft}}{\text{s}^2}$$

Problem 17-50

The dresser has a weight W and is pushed along the floor. If the coefficient of static friction at A and B is μ_s and the coefficient of kinetic friction is μ_k, determine the maximum horizontal force P that can be applied without causing the dresser to tip over.



Given:

$W = 80\ \text{lb}$

$\mu_s = 0.3$

$\mu_k = 0.2$

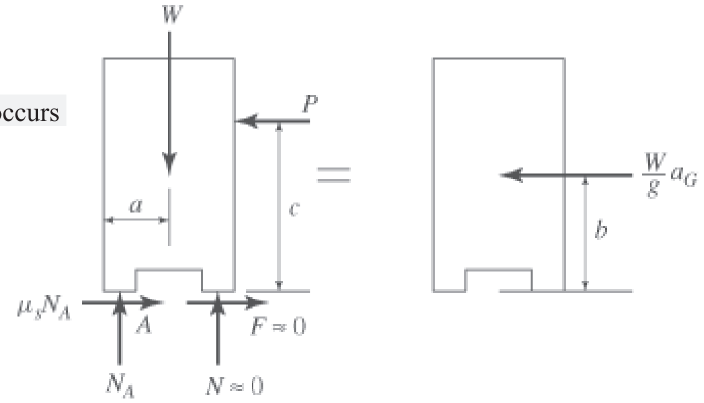
$a = 1.5\ \text{ft}$

$b = 2.5\ \text{ft}$

$c = 4\ \text{ft}$

Solution:

Dresser slides before tipping occurs



Guesses

$N_A = 1\ \text{lb}$

$P = 1\ \text{lb}$

$a_G = 1\ \frac{\text{ft}}{\text{s}^2}$

Given

$$N_A - W = 0 \qquad \mu_k N_A - P = \left(\frac{-W}{g}\right)a_G \qquad P(c-b) - N_A a + \mu_k N_A b = 0$$

$$\begin{pmatrix} N_A \\ P \\ a_G \end{pmatrix} = \text{Find}(N_A, P, a_G) \qquad a_G = 15.02\,\frac{\text{ft}}{\text{s}^2} \qquad N_A = 80\,\text{lb} \qquad P = 53.3\,\text{lb}$$

Problem 17-51

The crate C has weight W and rests on the truck elevator for which the coefficient of static friction is μ_s. Determine the largest initial angular acceleration α starting from rest, which the parallel links AB and DE can have without causing the crate to slip. No tipping occurs.

Given:

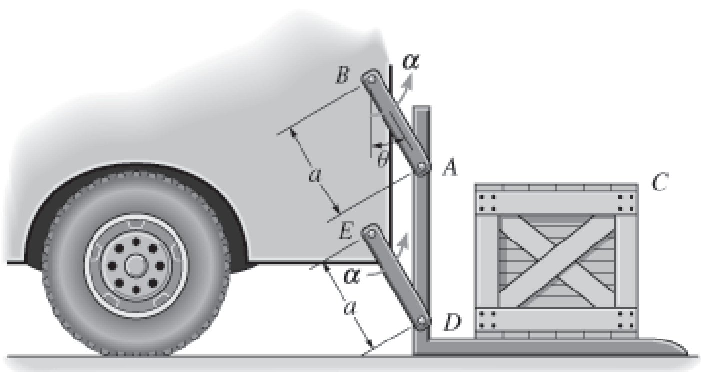
$W = 150\ \text{lb}$

$\mu_s = 0.4$

$a = 2\ \text{ft}$

$\theta = 30\ \text{deg}$

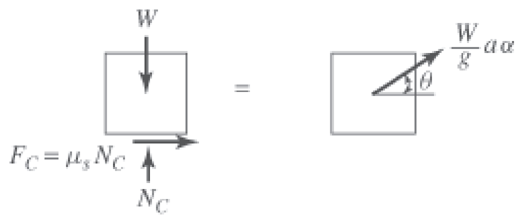
$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution:

Initial Guesses: $N_C = 1\ \text{lb} \qquad \alpha = 1\ \dfrac{\text{rad}}{\text{s}^2}$

Given



$$\mu_s N_C = \left(\frac{W}{g}\right)\alpha a\cos(\theta)$$

$$N_C - W = \left(\frac{W}{g}\right)\alpha(a)\sin(\theta)$$

$$\begin{pmatrix} N_C \\ \alpha \end{pmatrix} = \text{Find}(N_C, \alpha) \qquad N_C = 195.043\,\text{lb} \qquad \alpha = 9.669\,\frac{\text{rad}}{\text{s}^2}$$

***Problem 17-52**

The two rods EF and HI each of weight W are fixed (welded) to the link AC at E. Determine the normal force N_E, shear force V_E, and moment M_E, which the bar AC exerts on FE at E if at the instant θ link AB has an angular velocity ω and an angular acceleration α as shown.

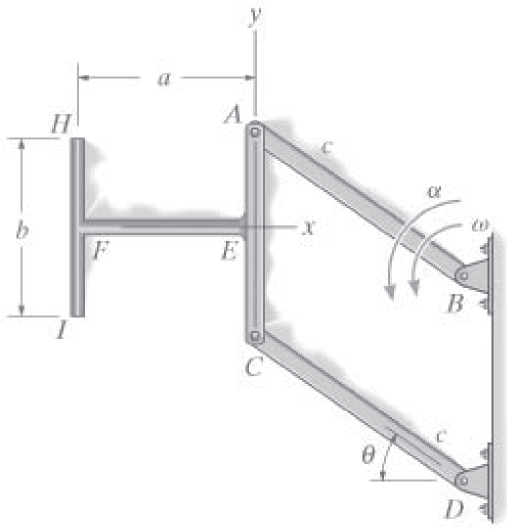
Given:

$W = 3\ \text{lb}$ $a = 2\ \text{ft}$ $g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$\theta = 30\ \text{deg}$ $b = 2\ \text{ft}$

$\omega = 5\ \frac{\text{rad}}{\text{s}}$ $c = 3\ \text{ft}$

$\alpha = 8\ \frac{\text{rad}}{\text{s}^2}$ $d = 3\ \text{ft}$



Solution:

$$x' = \frac{W a + W\left(\frac{a}{2}\right)}{2W}$$

$$a_{Gn} = c\omega^2$$

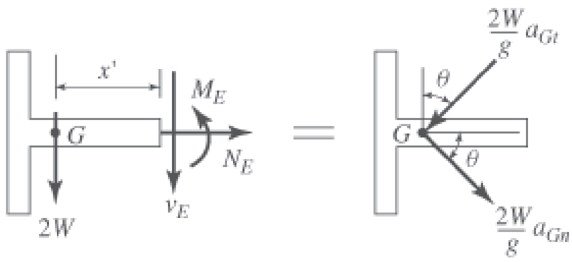
$$a_{Gt} = c\alpha$$

Guesses

$V_E = 1\ \text{lb}$

$N_E = 1\ \text{lb}$

$M_E = 1\ \text{lb}\cdot\text{ft}$



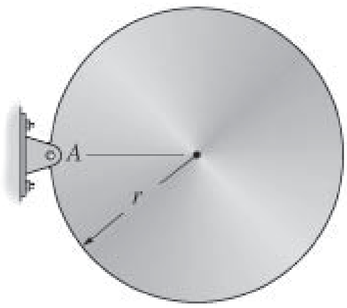
Given $$-2W - V_E = -2\left(\frac{W}{g}\right)a_{Gt}\cos(\theta) - 2\left(\frac{W}{g}\right)a_{Gn}\sin(\theta)$$

$$N_E = 2\left(\frac{W}{g}\right)a_{Gn}\cos(\theta) - 2\left(\frac{W}{g}\right)a_{Gt}\sin(\theta) \qquad M_E - V_E x' = 0$$

$$\begin{pmatrix} V_E \\ N_E \\ M_E \end{pmatrix} = \text{Find}(V_E, N_E, M_E) \qquad \begin{pmatrix} N_E \\ V_E \end{pmatrix} = \begin{pmatrix} 9.87 \\ 4.86 \end{pmatrix} \text{lb} \qquad M_E = 7.29\ \text{lb}\cdot\text{ft}$$

Problem 17-53

The disk of mass M is supported by a pin at A. If it is released from rest from the position shown, determine the initial horizontal and vertical components of reaction at the pin.

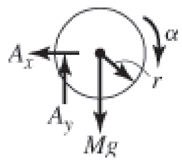


Given:

$M = 80\ \text{kg}$ $g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$r = 1.5\ \text{m}$

Solution: Guesses $A_x = 1\ \text{N}$ $A_y = 1\ \text{N}$ $\alpha = 1\ \dfrac{\text{rad}}{\text{s}^2}$



Given $Mgr = \dfrac{3}{2}Mr^2\alpha$ $-A_x = 0$ $A_y - Mg = -Mr\alpha$

$$\begin{pmatrix} A_x \\ A_y \\ \alpha \end{pmatrix} = \text{Find}(A_x, A_y, \alpha) \qquad \alpha = 4.36\ \frac{\text{rad}}{\text{s}^2} \qquad \begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 0 \\ 262 \end{pmatrix}\text{N}$$

Problem 17-54

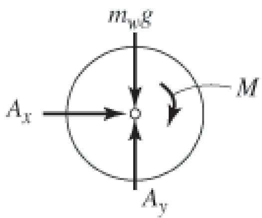
The wheel of mass m_w has a radius of gyration k_A. If the wheel is subjected to a moment $M = bt$, determine its angular velocity at time t starting from rest. Also, compute the reactions which the fixed pin A exerts on the wheel during the motion.

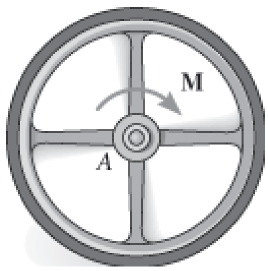
Given:

$m_W = 10\ \text{kg}$ $t = 3\ \text{s}$

$k_A = 200\ \text{mm}$ $g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

$b = 5\ \text{N}\ \dfrac{\text{m}}{\text{s}}$





Solution:

$bt = m_W k_A^2 \alpha$ $\alpha = \dfrac{bt}{m_W k_A^2}$

$\omega = \dfrac{bt^2}{2m_W k_A^2}$ $\omega = 56.2\ \dfrac{\text{rad}}{\text{s}}$

$A_x = 0\ \text{N}$ $A_y - m_W g = 0$

$A_y = m_W g$ $\begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 0 \\ 98.1 \end{pmatrix}\text{N}$

Problem 17-55

The fan blade has mass m_b and a moment of inertia I_0 about an axis passing through its center O. If it is subjected to moment $M = A(1 - e^{bt})$ determine its angular velocity when $t = t_1$ starting from rest.

Given:

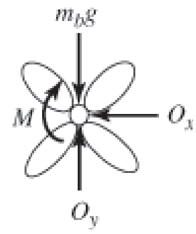
$m_b = 2\ \text{kg}$

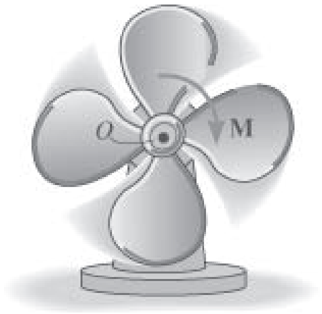
$I_O = 0.18\ \text{kg}\cdot\text{m}^2$

$A = 3\ \text{N}\cdot\text{m}$

$b = -0.2\ \text{s}^{-1}$

$t_1 = 4\ \text{s}$





Solution:

$$A\left(1 - e^{b\,t}\right) = I_O\alpha \qquad \alpha = \frac{A}{I_O}\left(1 - e^{b\,t_1}\right)$$

$$\omega = \frac{A}{I_O}\left(t_1 + \frac{1}{b} - \frac{1}{b}e^{b\,t_1}\right) \qquad \omega = 20.8\ \frac{\text{rad}}{\text{s}}$$

***Problem 17-56**

The rod of weight W is pin-connected to its support at A and has an angular velocity ω when it is in the horizontal position shown. Determine its angular acceleration and the horizontal and vertical components of reaction which the pin exerts on the rod at this instant.

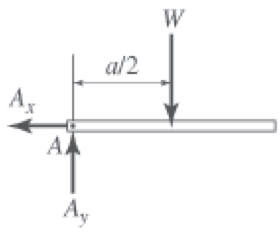
Given:

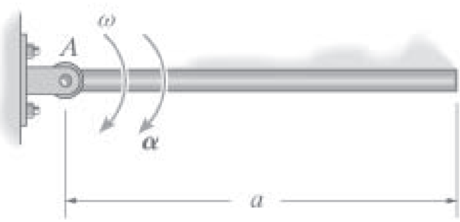
$\omega = 4\ \frac{\text{rad}}{\text{s}}$

$W = 10\ \text{lb}$

$a = 6\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$





Solution:

$$A_x = \left(\frac{W}{g}\right)\omega^2\left(\frac{a}{2}\right) \qquad A_x = 14.9\ \text{lb}$$

$$W\frac{a}{2} = \frac{1}{3}\left(\frac{W}{g}\right)a^2\alpha \qquad \alpha = \frac{3}{2a}g \qquad \alpha = 8.05\ \frac{\text{rad}}{\text{s}^2}$$

$$W - A_y = \left(\frac{W}{g}\right)\alpha\left(\frac{a}{2}\right)$$

$$A_y = W - \left(\frac{W}{g}\right)\alpha\left(\frac{a}{2}\right) \qquad A_y = 2.50\ \text{lb}$$

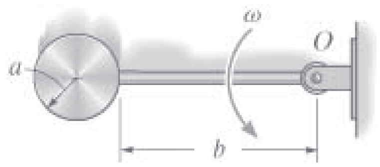
Problem 17-57

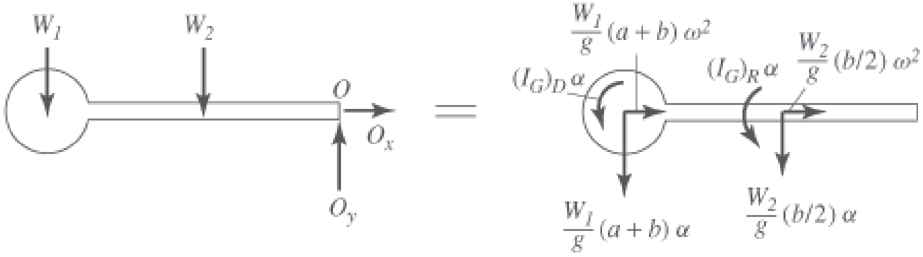
The pendulum consists of a disk of weight W_1 and a slender rod of weight W_2. Determine the horizontal and vertical components of reaction that the pin O exerts on the rod just as it passes the horizontal position, at which time its angular velocity is ω.

Given:

$W_1 = 15 \text{ lb} \quad a = 0.75 \text{ ft} \quad \omega = 8 \dfrac{\text{rad}}{\text{s}}$

$W_2 = 10 \text{ lb} \quad b = 3 \text{ ft}$





Solution: $$I_O = \frac{1}{2}\left(\frac{W_1}{g}\right)a^2 + \left(\frac{W_1}{g}\right)(a+b)^2 + \frac{1}{3}\left(\frac{W_2}{g}\right)b^2$$

Guesses $\alpha = 10 \dfrac{\text{rad}}{\text{s}^2} \qquad O_x = 100 \text{ lb} \qquad O_y = 5 \text{ lb}$

Given $$O_y - W_1 - W_2 = -\frac{W_1}{g}(a+b)\alpha - \left(\frac{W_2}{g}\right)\frac{b}{2}\alpha$$

$$O_x = \left(\frac{W_1}{g}\right)(a+b)\,\omega^2 + \left(\frac{W_2}{g}\right)\left(\frac{b}{2}\right)\omega^2$$

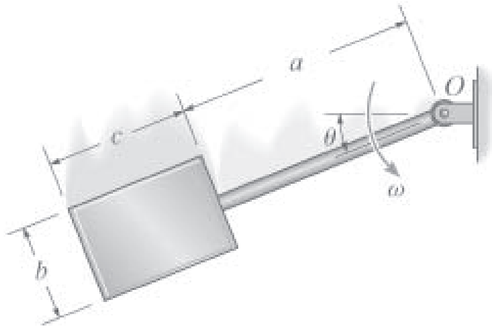
$$W_1(a+b) + W_2\left(\frac{b}{2}\right) = I_O\alpha$$

$$\begin{pmatrix}\alpha \\ O_x \\ O_y\end{pmatrix} = \text{Find}(\alpha, O_x, O_y) \qquad \alpha = 9.36 \frac{\text{rad}}{\text{s}^2} \qquad \begin{pmatrix}O_x \\ O_y\end{pmatrix} = \begin{pmatrix}141.61 \\ 4.29\end{pmatrix}\text{lb}$$

Problem 17-58

The pendulum consists of a uniform plate of mass M_1 and a slender rod of mass M_2. Determine the horizontal and vertical components of reaction that the pin O exerts on the rod at the instant shown at which time its angular velocity is ω.

Given:

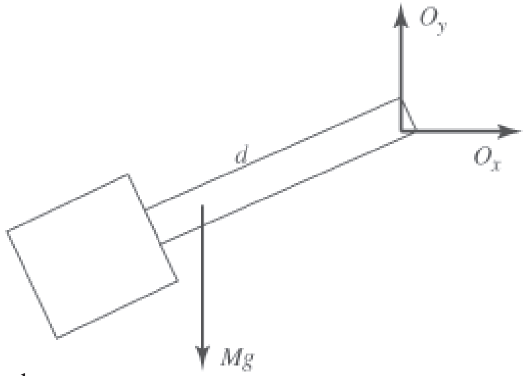


$M_1 = 5\ \text{kg}$ $\quad a = 0.5\ \text{m}$

$M_2 = 2\ \text{kg}$ $\quad b = 0.2\ \text{m}$

$\omega = 3\ \frac{\text{rad}}{\text{s}}$ $\quad c = 0.3\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$ $\quad \theta = 30\ \text{deg}$



Solution: $\quad M = M_1 + M_2$

$$I_O = \frac{1}{12}M_1\left(b^2 + c^2\right) + M_1\left(a + \frac{c}{2}\right)^2 + \frac{1}{3}M_2 a^2$$

$$d = \frac{M_1\left(a + \frac{c}{2}\right) + M_2\left(\frac{a}{2}\right)}{M}$$

Guesses $\quad O_x = 1\ \text{N} \quad O_y = 1\ \text{N} \quad \alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

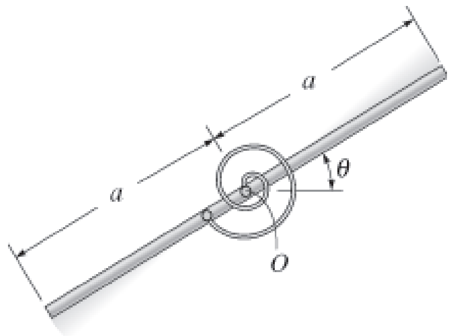
Given $\quad O_x = M d\alpha \sin(\theta) + M d\omega^2 \cos(\theta) \qquad M g d \cos(\theta) = I_O\alpha$

$$O_y - Mg = -M d\alpha\cos(\theta) + M d\omega^2\sin(\theta)$$

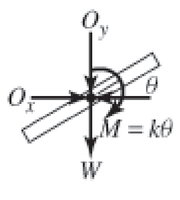
$$\begin{pmatrix} O_x \\ O_y \\ \alpha \end{pmatrix} = \text{Find}(O_x, O_y, \alpha) \qquad \alpha = 13.65\ \frac{\text{rad}}{\text{s}^2} \qquad \begin{pmatrix} O_x \\ O_y \end{pmatrix} = \begin{pmatrix} 54.8 \\ 41.2 \end{pmatrix}\ \text{N}$$

Problem 17-59

The bar of weight W is pinned at its center O and connected to a torsional spring. The spring has a stiffness k, so that the torque developed is $M = k\theta$. If the bar is released from rest when it is vertical at $\theta = 90°$, determine its angular velocity at the instant $\theta = 0°$.



Given:



$W = 10\ \text{lb}$

$k = 5\ \frac{\text{lb}\cdot\text{ft}}{\text{rad}}$

$a = 1 \text{ ft}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$

Solution:

$-k\theta = \frac{1}{12}\left(\frac{W}{g}\right)(2a)^2\alpha \qquad \alpha = \frac{-3kg}{Wa^2}\theta \qquad \frac{\omega^2}{2} - \frac{\omega_0^2}{2} = \frac{-3kg}{Wa^2}\left(\frac{\theta^2}{2} - \frac{\theta_0^2}{2}\right)$

$\omega = \sqrt{\frac{3kg}{Wa^2}\left(\frac{\pi}{2}\right)^2} \qquad \omega = 10.917 \frac{\text{rad}}{\text{s}}$

***Problem 17-60**

The bar of weight *w* is pinned at its center *O* and connected to a torsional spring. The spring has a stiffness *k*, so that the torque developed is $M = k\theta$. If the bar is released from rest when it is vertical at $\theta = 90°$, determine its angular velocity at the instant $\theta = \theta_1$.

Given:

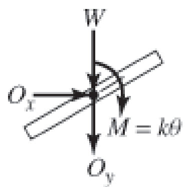
$W = 10 \text{ lb}$

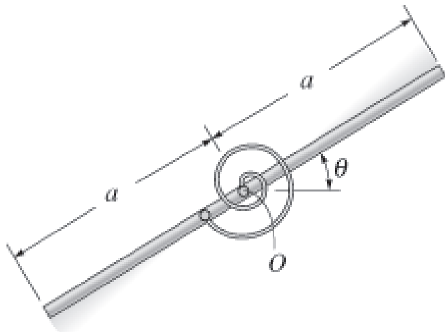
$k = 5 \frac{\text{lb} \cdot \text{ft}}{\text{rad}}$

$a = 1 \text{ ft}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$

$\theta_1 = 45 \text{ deg}$





Solution:

$-k\theta = \frac{1}{12}\left(\frac{W}{g}\right)(2a)^2\alpha \qquad \alpha = \frac{-3kg}{Wa^2}\theta$

$\frac{\omega^2}{2} - \frac{\omega_0^2}{2} = \frac{-3kg}{Wa^2}\left(\frac{\theta^2}{2} - \frac{\theta_0^2}{2}\right)$

$\omega = \sqrt{\frac{3kg}{Wa^2}\left[(90 \text{ deg})^2 - \theta_1^2\right]} \qquad \omega = 9.454 \frac{\text{rad}}{\text{s}}$

Problem 17-61

The roll of paper of mass M has radius of gyration k_A about an axis passing through point A. It is pin-supported at both ends by two brackets AB. If the roll rests against a wall for which the coefficient of kinetic friction is μ_k and a vertical force F is applied to the end of the paper, determine the angular acceleration of the roll as the paper unrolls.

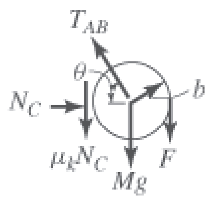
Given:

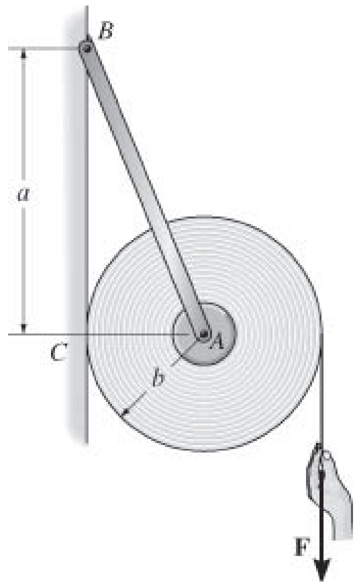
$M = 20\ \text{kg}$ $\quad a = 300\ \text{mm}$

$k_A = 90\ \text{mm}$ $\quad b = 125\ \text{mm}$

$\mu_k = 0.2$ $\quad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

$F = 30\ \text{N}$





Solution: $\quad \theta = \operatorname{atan}\left(\dfrac{a}{b}\right)$

Guesses $\quad N_C = 1\ \text{N} \quad \alpha = 1\ \dfrac{\text{rad}}{\text{s}^2} \quad T_{AB} = 1\ \text{N}$

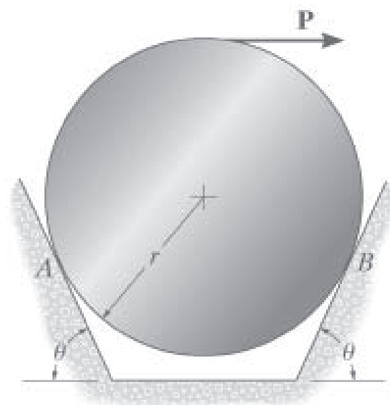
Given $\quad N_C - T_{AB}\cos(\theta) = 0$

$$T_{AB}\sin(\theta) - \mu_k N_C - M g - F = 0$$

$$F b - \mu_k N_C b = M k_A^2 \alpha$$

$$\begin{pmatrix} N_C \\ \alpha \\ T_{AB} \end{pmatrix} = \operatorname{Find}(N_C, \alpha, T_{AB}) \qquad \begin{pmatrix} N_C \\ T_{AB} \end{pmatrix} = \begin{pmatrix} 102.818 \\ 267.327 \end{pmatrix} \text{N} \qquad \alpha = 7.281\ \frac{\text{rad}}{\text{s}^2}$$

Problem 17-62



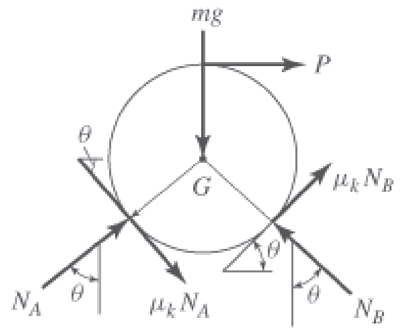
The cylinder has a radius r and mass m and rests in the trough for which the coefficient of kinetic friction at A and B is μ_k. If a horizontal force **P** is applied to the cylinder, determine the cylinder's angular acceleration when it begins to spin.

Solution:

$$P - (N_B - N_A)\sin(\theta) + \mu_k(N_A + N_B)\cos(\theta) = 0$$

$$(N_A + N_B)\cos(\theta) + \mu_k(N_B - N_A)\sin(\theta) - m g = 0$$

$$\left[\mu_k\left(N_A + N_B\right) - P\right]r = \frac{-1}{2} m r^2 \alpha$$



Solving

$$N_A + N_B = \frac{m g - \mu_k P}{\cos(\theta)\left(1 + \mu_k^2\right)}$$

$$N_B - N_A = \frac{\mu_k m g + P}{\sin(\theta)\left(1 + \mu_k^2\right)}$$

$$\alpha = \frac{-2\mu_k}{m r}\left[\frac{m g - \mu_k P}{\cos(\theta)\left(1 + \mu_k^2\right)}\right] + \frac{2P}{m r}$$

Problem 17-63

The uniform slender rod has a mass M. If the cord at A is cut, determine the reaction at the pin O, (a) when the rod is still in the horizontal position, and (b) when the rod swings to the vertical position.

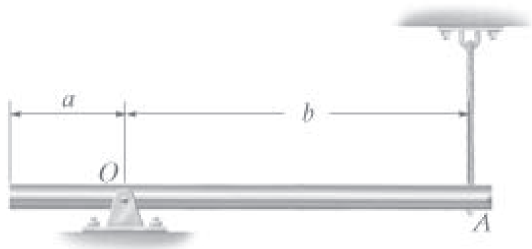
Given:

$M = 5 \text{ kg}$

$a = 200 \text{ mm}$

$b = 600 \text{ mm}$

$g = 9.81 \frac{\text{m}}{\text{s}^2}$



Solution: $L = a + b$ $\qquad d = \frac{b-a}{2}$ $\qquad I_O = \frac{1}{12} M L^2 + M d^2$

(a) In the horizontal position

Guesses $\quad a_G = 1 \frac{\text{m}}{\text{s}^2} \qquad \alpha = 1 \frac{\text{rad}}{\text{s}^2} \qquad O_x = 1 \text{ N} \qquad O_y = 1 \text{ N}$

Given $\quad a_G = \alpha d \qquad M g d = I_O \alpha \qquad -O_x = 0 \qquad O_y - M g = -M a_G$

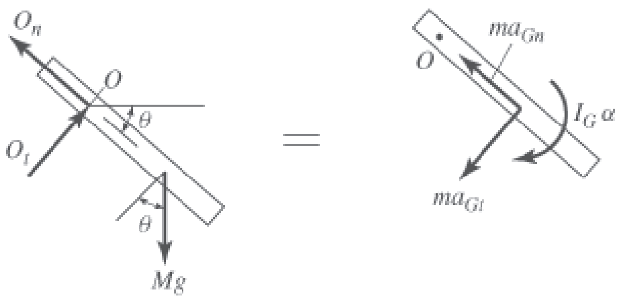
$$\begin{pmatrix} O_x \\ O_y \\ a_G \\ \alpha \end{pmatrix} = \text{Find}(O_x, O_y, a_G, \alpha) \quad a_G = 4.20 \frac{\text{m}}{\text{s}^2} \quad \alpha = 21.0 \frac{\text{rad}}{\text{s}^2} \quad \begin{pmatrix} O_x \\ O_y \end{pmatrix} = \begin{pmatrix} 0.0 \\ 28.0 \end{pmatrix} \text{N}$$

$$\left| \begin{pmatrix} O_x \\ O_y \end{pmatrix} \right| = 28.0 \text{ N}$$

Next examine a general position

$$M g d \cos(\theta) = I_O \alpha$$

$$\alpha = \frac{M g d}{I_O} \cos(\theta)$$



$$\frac{\omega^2}{2} = \frac{M g d}{I_O} \sin(\theta)$$

$$\omega = \sqrt{\frac{2 M g d}{I_O} \sin(\theta)}$$

(b) In the vertical position (θ= 90 deg) $\quad \omega = \sqrt{\frac{2 M g d}{I_O} \sin(90 \text{ deg})}$

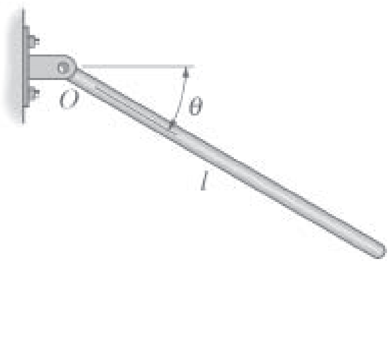
Guesses $\quad \alpha = 1 \frac{\text{rad}}{\text{s}^2} \quad O_x = 1 \text{ N} \quad O_y = 1 \text{ N}$

Given $\quad 0 = I_O \alpha \quad -O_x = -M \alpha d \quad O_y - M g = M d \omega^2$

$$\begin{pmatrix} \alpha \\ O_x \\ O_y \end{pmatrix} = \text{Find}(\alpha, O_x, O_y) \quad \alpha = 0.0 \frac{\text{rad}}{\text{s}^2} \quad \begin{pmatrix} O_x \\ O_y \end{pmatrix} = \begin{pmatrix} 0.0 \\ 91.1 \end{pmatrix} \text{N} \quad \left| \begin{pmatrix} O_x \\ O_y \end{pmatrix} \right| = 91.1 \text{ N}$$

***Problem 17-64**

The bar has a mass m and length l. If it is released from rest from the position shown, determine its angular acceleration and the horizontal and vertical components of reaction at the pin O.



Given:

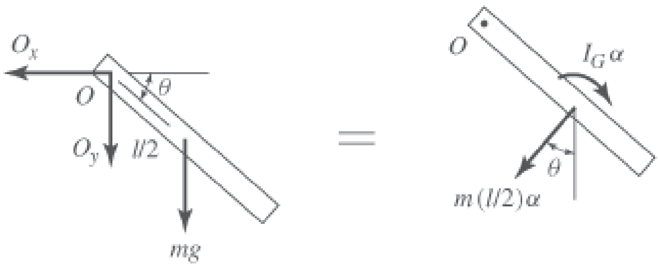
$\theta = 30 \text{ deg}$

Solution:

$$m g \frac{l}{2} \cos(\theta) = \frac{1}{3} m l^2 \alpha$$

$$O_x = m\frac{l}{2}\alpha\sin(\theta)$$

$$O_y - mg = -m\frac{l}{2}\alpha\cos(\theta)$$



Solving

$$\alpha = \frac{3g}{2l}\cos(\theta) \qquad O_x = \frac{3mg}{8}\sin(2\theta) \qquad O_y = mg\left(1 - \frac{3}{4}\cos(\theta)^2\right)$$

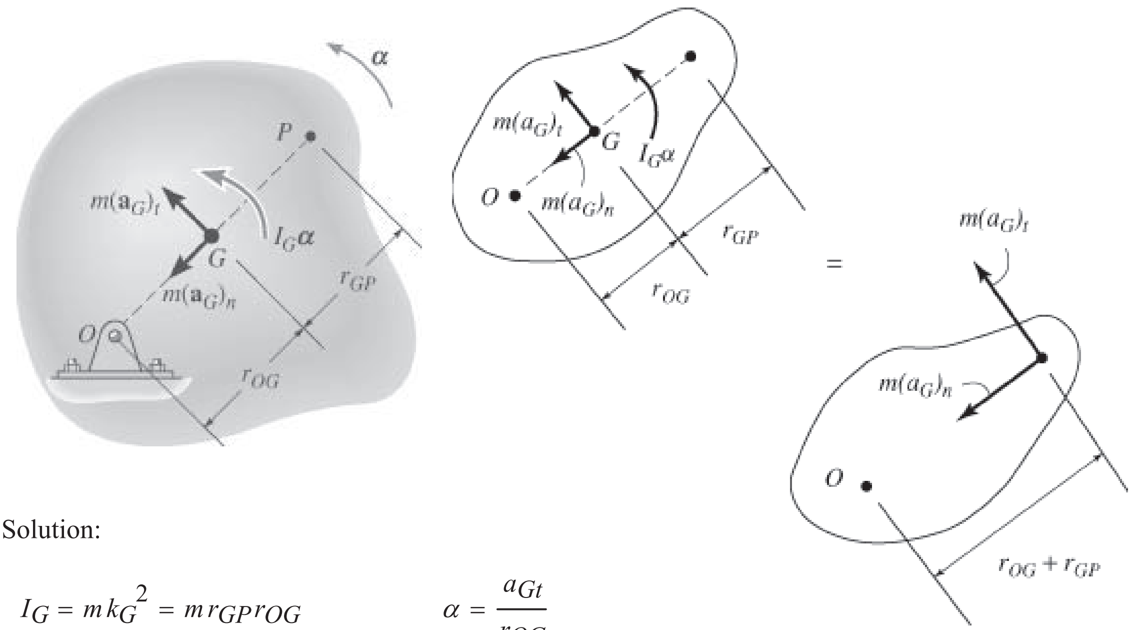
$$k_1 = \frac{3}{2}\cos(\theta) \qquad k_2 = \frac{3}{8}\sin(2\theta) \qquad k_3 = 1 - \frac{3}{4}\cos(\theta)^2$$

$$\alpha = k_1\frac{g}{l} \qquad O_x = k_2 mg \qquad O_y = k_3 mg$$

$k_1 = 1.30$ $\qquad$ $k_2 = 0.325$ $\qquad$ $k_3 = 0.437$

Problem 17-65

The kinetic diagram representing the general rotational motion of a rigid body about a fixed axis at O is shown in the figure. Show that $I_G\alpha$ may be eliminated by moving the vectors $m(a_{Gt})$ and $m(a_{Gn})$ to point P, located a distance $r_{GP} = k_G^2/r_{OG}$ from the center of mass G of the body. Here k_G represents the radius of gyration of the body about G. The point P is called the *center of percussion* of the body.



Solution:

$$I_G = mk_G^2 = mr_{GP}r_{OG} \qquad \alpha = \frac{a_{Gt}}{r_{OG}}$$

$$m\,a_{Gt}r_{OG} + I_G\alpha = m\,a_{Gt}r_{OG} + (m\,r_{OG}r_{GP})\left(\frac{a_{Gt}}{r_{OG}}\right)$$

$$m\,a_{Gt}r_{OG} + I_G\alpha = m\,a_{Gt}(r_{OG} + r_{GP})$$ Q.E.D.

Problem 17-66

Determine the position of the center of percussion P of the slender bar of weight W. (See Prob. 17-65.) What is the horizontal force at the pin when the bar is struck at P with force F?

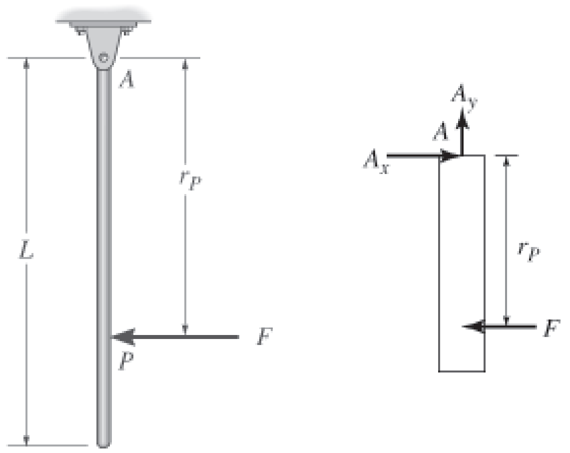
Given:

$W = 10\ \text{lb}$

$F = 20\ \text{lb}$

$L = 4\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution: $k_G = \dfrac{L}{\sqrt{12}}$

From Prob 17-65

$$r_p = \frac{L}{2} + \frac{k_G^2}{\frac{L}{2}} \qquad r_p = 2.667\ \text{ft}$$

Guesses $A_x = 1\ \text{lb}$ $\alpha = 1\ \dfrac{\text{rad}}{\text{s}^2}$

Given $A_x - F = \left(\dfrac{-W}{g}\right)\alpha\left(\dfrac{L}{2}\right)$ $-F\,r_p = \dfrac{-1}{3}\left(\dfrac{W}{g}\right)L^2\alpha$

$$\begin{pmatrix} A_x \\ \alpha \end{pmatrix} = \text{Find}(A_x, \alpha) \qquad \alpha = 32.2\ \frac{\text{rad}}{\text{s}^2} \qquad A_x = 2.35 \times 10^{-14}\ \text{lb}$$

A zero horizontal force is the condition used to define the center of percussion.

Problem 17-67

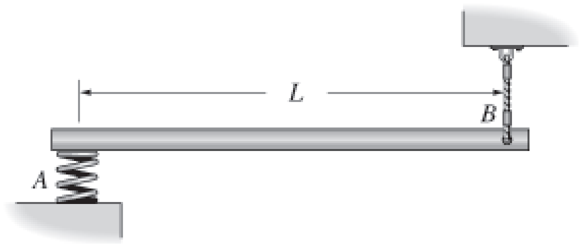
The slender rod of mass M is supported horizontally by a spring at A and a cord at B. Determine the angular acceleration of the rod and the acceleration of the rod's mass center at the instant the cord at B is cut. *Hint*: The stiffness of the spring is not needed for the calculation.

Given:

$M = 4\ \text{kg}$

$g = 9.81\ \frac{m}{s^2}$

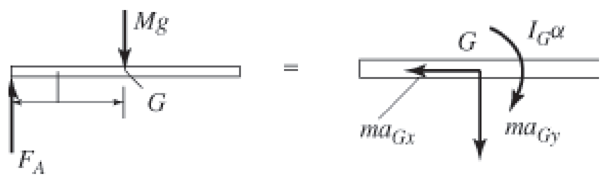
$L = 2\ m$



Solution:

Since the deflection of the spring is unchanged, we have

$$F_A = \frac{Mg}{2}$$



$$F_A \frac{L}{2} = \frac{1}{12} M L^2 \alpha$$

$$\alpha = \frac{6F_A}{ML} \qquad \alpha = 14.7\ \frac{rad}{s^2}$$

$$F_A - Mg = -Ma_{Gy} \qquad a_{Gy} = g - \frac{F_A}{M} \qquad a_{Gy} = 4.91\ \frac{m}{s^2} \qquad a_{Gx} = 0\ \frac{m}{s^2}$$

***Problem 17-68**

In order to experimentally determine the moment of inertia I_G of a connecting rod of mass M, the rod is suspended horizontally at A by a cord and at B by a bearing and piezoelectric sensor, an instrument used for measuring force. Under these equilibrium conditions, the force at B is measured as F_1. If, at the instant the cord is released, the reaction at B is measured as F_2, determine the value of I_G. The support at B does not move when the measurement is taken. For the calculation, the horizontal location of G must be determined.

Given:

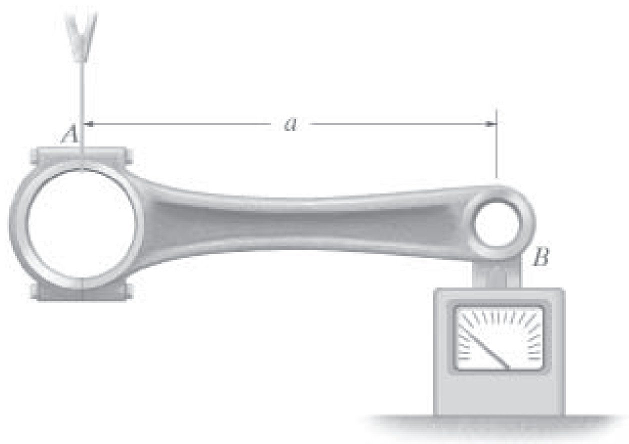
$M = 4\ kg$

$F_1 = 14.6\ N$

$F_2 = 9.3\ N$

$a = 350\ mm$

$g = 9.81\ \frac{m}{s^2}$



Solution:

Guesses

$x = 1\ mm \quad I_G = 1\ kg \cdot m^2 \quad A_y = 1\ N \quad \alpha = 1\ \frac{rad}{s^2}$

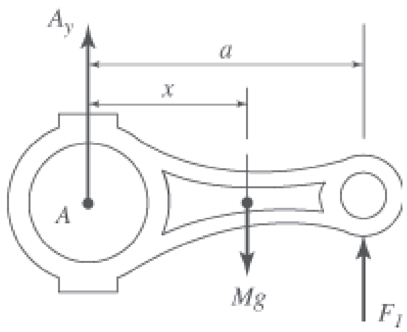
Given

$$A_y - Mg + F_1 = 0$$

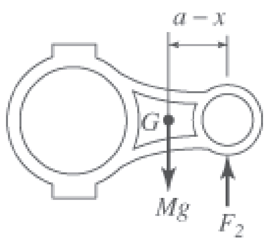
$$-Mgx + F_1 a = 0$$

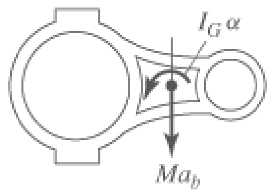
$$F_2 - Mg = -M\alpha(a - x)$$

$$Mg(a - x) = \left[I_G + M(a - x)^2\right]\alpha$$



$$\begin{pmatrix} x \\ I_G \\ A_y \\ \alpha \end{pmatrix} = \mathrm{Find}\left(x, I_G, A_y, \alpha\right)$$





$x = 130\,\text{mm}$ $\quad A_y = 24.6\ \text{N}$ $\quad \alpha = 34.1\ \dfrac{\text{rad}}{\text{s}^2}$ $\quad I_G = 0.0600\ \text{kg}\cdot\text{m}^2$

Problem 17-69

Disk D of weight W is subjected to counterclockwise moment $M = bt$. Determine the angular velocity of the disk at time t after the moment is applied. Due to the spring the plate P exerts constant force P on the disk. The coefficients of static and kinetic friction between the disk and the plate are μ_s and μ_k respectively. *Hint:* First find the time needed to start the disk rotating.

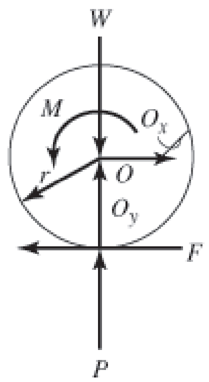
Given:

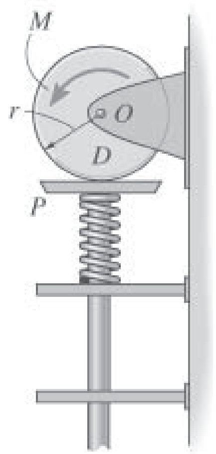
$W = 10\ \text{lb}$ $\qquad \mu_s = 0.3$

$b = 10\ \dfrac{\text{lb}\cdot\text{ft}}{\text{s}}$ $\qquad \mu_k = 0.2$

$t = 2\ \text{s}$ $\qquad r = 0.5\ \text{ft}$

$P = 100\ \text{lb}$ $\qquad g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$





Solution: When motion begins

$$b t_1 = \mu_s P r \qquad t_1 = \frac{\mu_s P r}{b} \qquad t_1 = 1.5\ \text{s}$$

At a later time we have

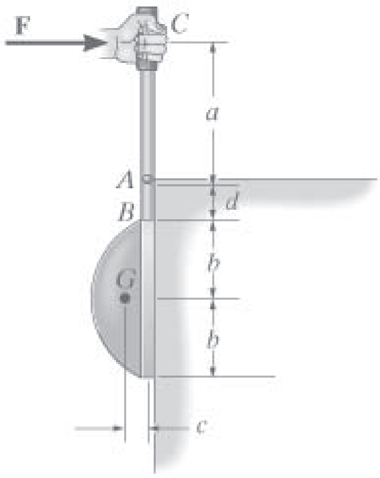
$$b t - \mu_k P r = \frac{1}{2}\left(\frac{W}{g}\right) r^2 \alpha \qquad \alpha = \frac{2g}{W r^2}\left(b t - \mu_k P r\right)$$

$$\omega = \frac{2g}{W r^2}\left[\frac{b}{2}\left(t^2 - t_1{}^2\right) - \mu_k P r\left(t - t_1\right)\right] \qquad \omega = 96.6\,\frac{\text{rad}}{\text{s}}$$

Problem 17-70

The furnace cover has a mass M and a radius of gyration k_G about its mass center G. If an operator applies a force F to the handle in order to open the cover, determine the cover's initial angular acceleration and the horizontal and vertical components of reaction which the pin at A exerts on the cover at the instant the cover begins to open. Neglect the mass of the handle BAC in the calculation.

Given:



$M = 20$ kg $\quad a = 0.7$ m

$k_G = 0.25$ m $\quad b = 0.4$ m

$F = 120$ N $\quad c = 0.25$ m

$d = 0.2$ m

Solution: $\quad \theta = \operatorname{atan}\left(\frac{c}{b + d}\right)$

Guesses $\quad \alpha = 5\,\frac{\text{rad}}{\text{s}^2}$

$A_x = 50$ N

$A_y = 20$ N

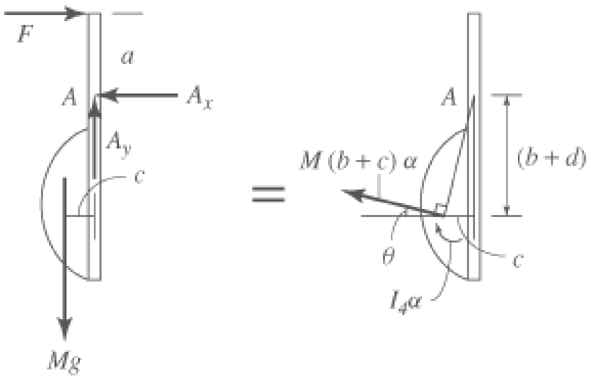
Given

$$A_x - F = M(c + b)\,\alpha\cos(\theta)$$

$$A_y - Mg = M(c + b)\,\alpha\sin(\theta)$$

$$F a - M g c = M c^2\alpha + M(c + b)^2\alpha$$

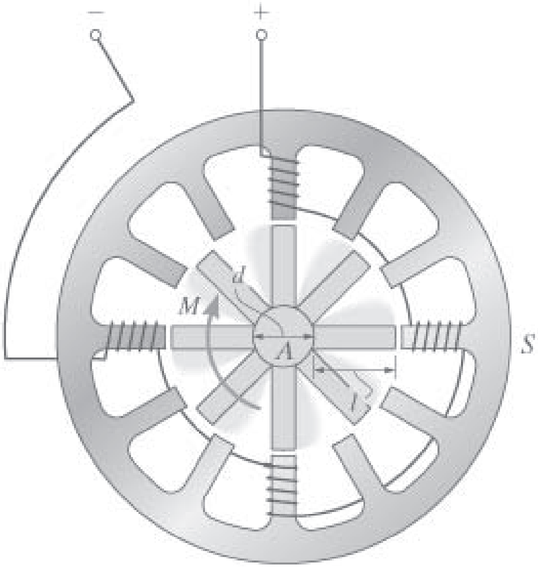
$$\begin{pmatrix}\alpha \\ A_x \\ A_y\end{pmatrix} = \text{Find}(\alpha, A_x, A_y)$$



$$\alpha = 3.60\,\frac{\text{rad}}{\text{s}^2} \qquad \begin{pmatrix}A_x \\ A_y\end{pmatrix} = \begin{pmatrix}163 \\ 214\end{pmatrix}\text{N}$$

Problem 17-71

The variable-resistance motor is often used for appliances, pumps, and blowers. By applying a current through the stator S, an electromagnetic field is created that "pulls in" the nearest rotor poles. The result of this is to create a torque M about the bearing at A. If the rotor is made from iron and has a cylindrical core of mass M_1, diameter d and eight extended slender rods, each having a mass M_2 and length l, determine its angular velocity at time t starting from rest.



Given:

$M_1 = 3\ \text{kg}$ $\quad l = 100\text{mm}$

$M_2 = 1\ \text{kg}$ $\quad d = 50\ \text{mm}$

$M = 4\ \text{N}\cdot\text{m}$ $\quad t = 5\ \text{s}$

Solution:

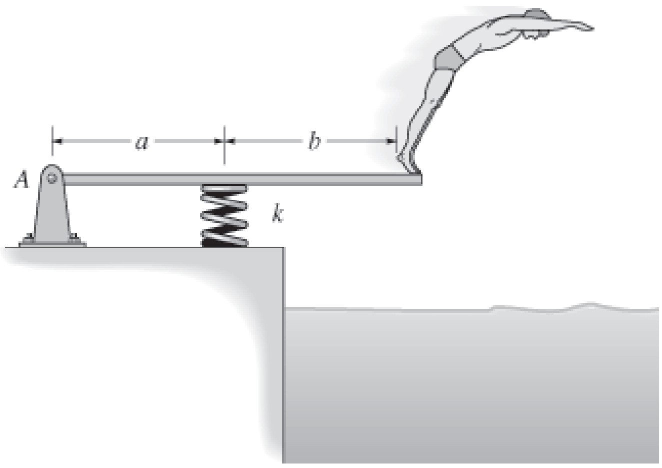
$$I_A = \frac{1}{2}M_1\left(\frac{d}{2}\right)^2 + 8\left[\frac{1}{12}M_2 l^2 + M_2\left(\frac{d}{2} + \frac{l}{2}\right)^2\right]$$

$$M = I_A\alpha \qquad \alpha = \frac{M}{I_A}$$

$$\omega = \alpha t \qquad \omega = 380\ \frac{\text{rad}}{\text{s}}$$

***Problem 17-72**

Determine the angular acceleration of the diving board of mass M and the horizontal and vertical components of reaction at the pin A the instant the man jumps off. Assume that the board is uniform and rigid, and that at the instant he jumps off the spring is compressed a maximum amount δ and the board is horizontal.



Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

$M = 25\ \text{kg}$

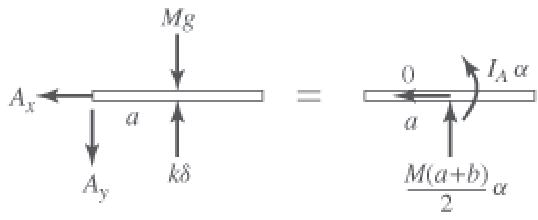
$\delta = 200\ \text{mm}$

$k = 7\ \frac{\text{kN}}{\text{m}}$

$a = 1.5\ \text{m}$

$b = 1.5\ \text{m}$

$g = 9.815\ \frac{\text{m}}{\text{s}^2}$



Solution:

Guesses $A_x = 1\ \text{N}$ $A_y = 1\ \text{N}$ $\alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

Given $-Mga + k\delta a = \frac{1}{3}M(a+b)^2\alpha$

$A_x = 0$ $-A_y - Mg + k\delta = M\alpha\left(\frac{a+b}{2}\right)$

$\begin{pmatrix} A_x \\ A_y \\ \alpha \end{pmatrix} = \text{Find}(A_x, A_y, \alpha)$ $\alpha = 23.1\ \frac{\text{rad}}{\text{s}^2}$ $\begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 0 \\ 289 \end{pmatrix}\ \text{N}$

Problem 17-73

The disk has mass M and is originally spinning at the end of the strut with angular velocity ω. If it is then placed against the wall, for which the coefficient of kinetic friction is μ_k, determine the time required for the motion to stop. What is the force in strut BC during this time?

Given:

$M = 20\ \text{kg}$

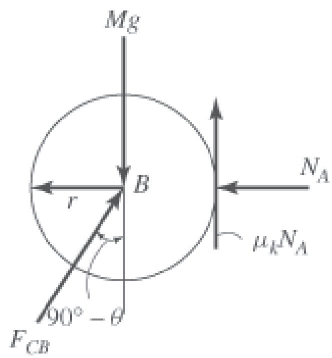
$\omega = 60\ \frac{\text{rad}}{\text{s}}$

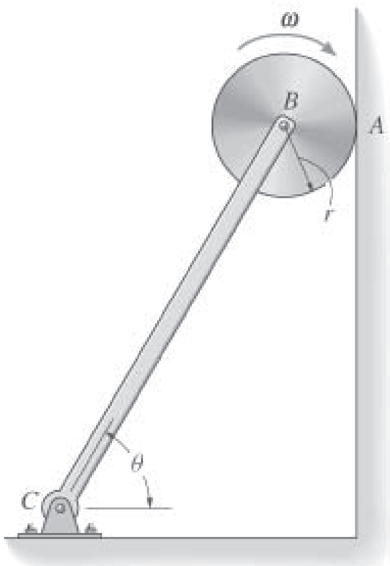
$\mu_k = 0.3$

$\theta = 60\ \text{deg}$

$r = 150\ \text{mm}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$





Solution:

Initial Guess:

$F_{CB} = 1\ \text{N}$ $\alpha = 1\ \frac{\text{rad}}{\text{s}^2}$ $N_A = 1\ \text{N}$

Given $F_{CB}\cos(\theta) - N_A = 0$

$F_{CB}\sin(\theta) - Mg + \mu_k N_A = 0$

$\mu_k N_A r = \frac{1}{2} M r^2 \alpha$

$$\begin{pmatrix} F_{CB} \\ N_A \\ \alpha \end{pmatrix} = \text{Find}(F_{CB}, N_A, \alpha) \qquad \alpha = 19.311 \frac{\text{rad}}{\text{s}^2} \qquad N_A = 96.6\ \text{N} \qquad F_{CB} = 193\ \text{N}$$

$$t = \frac{\omega}{\alpha} \qquad t = 3.107\ \text{s}$$

Problem 17-74

The relay switch consists of an electromagnet E and an armature AB (slender bar) of mass M which is pinned at A and lies in the vertical plane.When the current is turned off, the armature is held open against the smooth stop at B by the spring CD, which exerts an upward vertical force F_s on the armature at C. When the current is turned on, the electromagnet attracts the armature at E with a vertical force F. Determine the initial angular acceleration of the armature when the contact BF begins to close.

Given:

$M = 20\ \text{gm}$

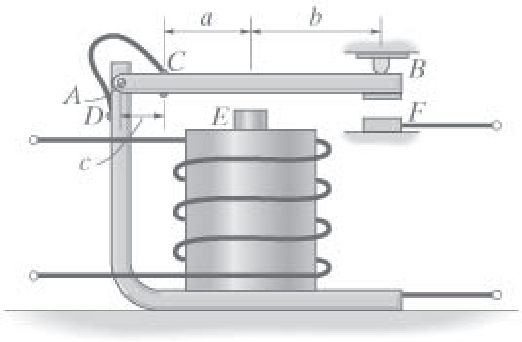
$F = 0.8\ \text{N}$

$F_s = 0.85\ \text{N}$

$a = 20\ \text{mm}$

$b = 30\ \text{mm}$

$c = 10\ \text{mm}$

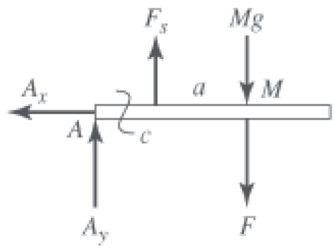


Solution:

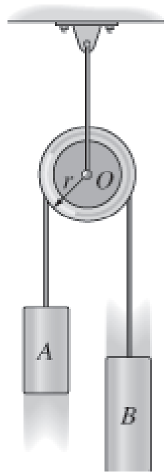
$$F_s c - Mg\left(\frac{a+b+c}{2}\right) - F(a+c) = -\frac{1}{3} M(a+b+c)^2 \alpha$$

$$\alpha = 3\left[\frac{Mg\left(\frac{a+b+c}{2}\right) + F(a+c) - F_s c}{M(a+b+c)^2}\right]$$

$$\alpha = 891 \frac{\text{rad}}{\text{s}^2}$$



Problem 17-75



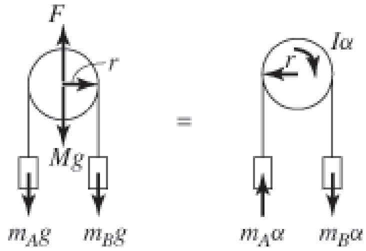
The two blocks A and B have a mass m_A and m_B, respectively, where $m_B > m_A$. If the pulley can be treated as a disk of mass M, determine the acceleration of block A. Neglect the mass of the cord and any slipping on the pulley.

Solution:

$$a = \alpha r$$

$$m_B g r - m_A g r = \frac{1}{2} M r^2 \alpha + m_B r^2 \alpha + m_A r^2 \alpha$$

$$\alpha = \frac{g(m_B - m_A)}{r\left(\frac{1}{2}M + m_B + m_A\right)}$$

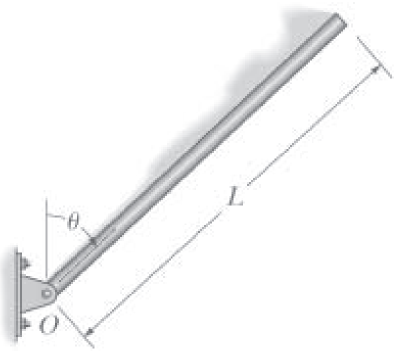


$$a = \frac{g(m_B - m_A)}{\frac{1}{2}M + m_B + m_A}$$

***Problem 17-76**

The rod has a length L and mass m. If it is released from rest when $\theta = 0°$, determine its angular velocity as a function of θ. Also, express the horizontal and vertical components of reaction at the pin O as a function of θ.

Solution:

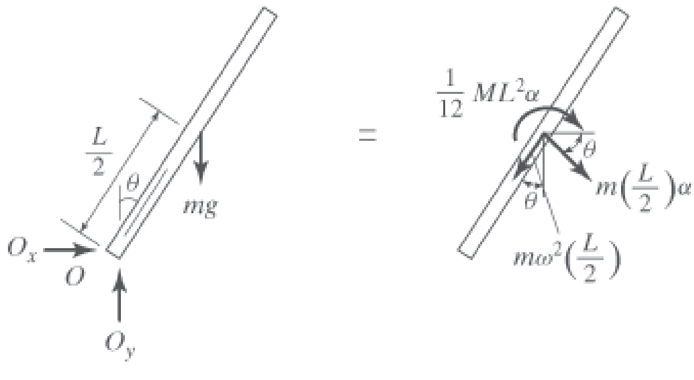


$$mg\frac{L}{2}\sin(\theta) = \frac{1}{3}mL^2\alpha$$

$$\alpha = \frac{3g}{2L}\sin(\theta)$$

$$\frac{\omega^2}{2} = \frac{3g}{2L}(1 - \cos(\theta))$$

$$\omega = \sqrt{\frac{3g}{L}(1 - \cos(\theta))}$$



$$O_x = m\frac{L}{2}\alpha\cos(\theta) - m\frac{L}{2}\omega^2\sin(\theta)$$

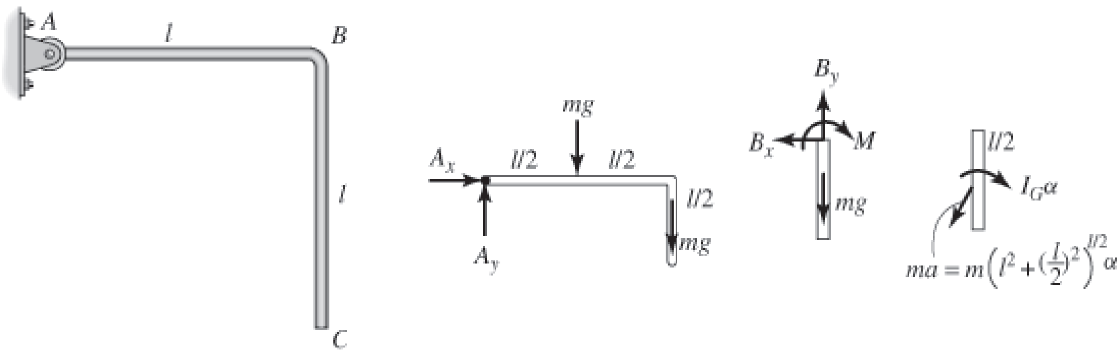
$$O_x = mg\sin(\theta)\left(\frac{9}{4}\cos(\theta) - \frac{3}{2}\right)$$

$$O_y - m\,g = -m\left(\frac{L}{2}\right)\alpha\sin(\theta) - m\left(\frac{L}{2}\right)\omega^2\cos(\theta)$$

$$O_y = m\,g\left(1 - \frac{3}{2}\cos(\theta) + \frac{3}{2}\cos(\theta)^2 - \frac{3}{4}\sin(\theta)^2\right)$$

Problem 17-77

The two-bar assembly is released from rest in the position shown. Determine the initial bending moment at the fixed joint B. Each bar has mass m and length l.



Solution:

$$I_A = \frac{1}{3}m\,l^2 + \frac{1}{12}m\,l^2 + m\left[l^2 + \left(\frac{l}{2}\right)^2\right] = \frac{5}{3}m\,l^2$$

$$m\,g\frac{l}{2} + m\,g\,l = I_A\alpha \qquad \alpha = \frac{9}{10}\frac{g}{l}$$

$$M = \frac{1}{12}m\,l^2\alpha + m\left(\frac{l}{2}\right)\alpha\left(\frac{l}{2}\right) = \frac{1}{3}m\,l^2\alpha \qquad M_A = \frac{3}{10}m\,g\,l$$

Problem 17-78

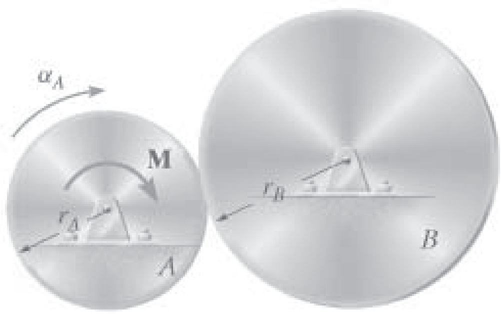
Disk A has weight W_A and disk B has weight W_B. If no slipping occurs between them, determine the couple moment M which must be applied to disk A to give it an angular acceleration α_A.

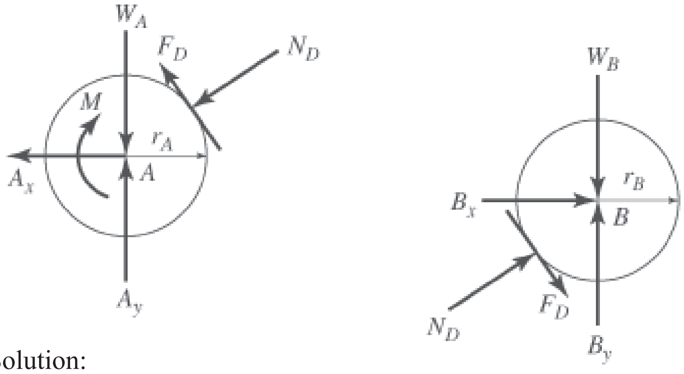
Given:

$W_A = 5\text{ lb}$ $\quad r_A = 0.5\text{ ft}$

$W_B = 10\text{ lb}$ $\quad r_B = 0.75\text{ ft}$

$\alpha_A = 4\ \dfrac{\text{rad}}{\text{s}^2}$





Solution:

Guesses $\alpha_B = 1\ \frac{rad}{s^2}$ $M = 1\ lb\cdot ft$ $F_D = 1\ lb$

Given

$$M - F_D r_A = \frac{1}{2}\left(\frac{W_A}{g}\right) r_A^{\ 2}\alpha_A \qquad F_D r_B = \frac{1}{2}\left(\frac{W_B}{g}\right) r_B^{\ 2}\alpha_B \qquad r_A\alpha_A = r_B\alpha_B$$

$$\begin{pmatrix} M \\ \alpha_B \\ F_D \end{pmatrix} = \text{Find}(M, \alpha_B, F_D) \qquad \alpha_B = 2.67\ \frac{rad}{s^2} \qquad F_D = 0.311\ lb \qquad M = 0.233\ lb\cdot ft$$

Problem 17-79

The wheel has mass M and radius of gyration k_B. It is originally spinning with angular velocity ω_1. If it is placed on the ground, for which the coefficient of kinetic friction is μ_c, determine the time required for the motion to stop. What are the horizontal and vertical components of reaction which the pin at A exerts on AB during this time? Neglect the mass of AB.

Given:

$M = 25\ kg$

$k_B = 0.15\ m$

$\omega_1 = 40\ \frac{rad}{s}$

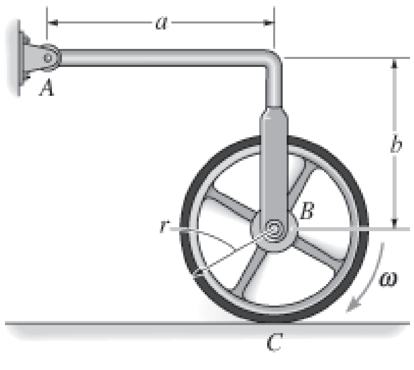
$\mu_C = 0.5$

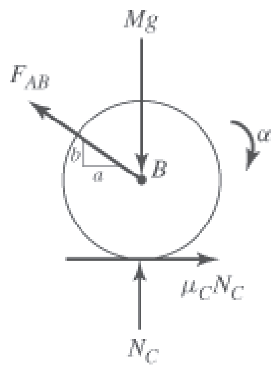
$a = 0.4\ m$

$b = 0.3\ m$

$r = 0.2\ m$

$g = 9.81\ \frac{m}{s^2}$





Solution: Guesses $F_{AB} = 1\ \text{N}$ $N_C = 1\ \text{N}$ $\alpha = 1\ \dfrac{\text{rad}}{\text{s}^2}$

Given

$$\mu_C N_C - \left(\frac{a}{\sqrt{a^2 + b^2}}\right) F_{AB} = 0$$

$$N_C - Mg + \left(\frac{b}{\sqrt{a^2 + b^2}}\right) F_{AB} = 0$$

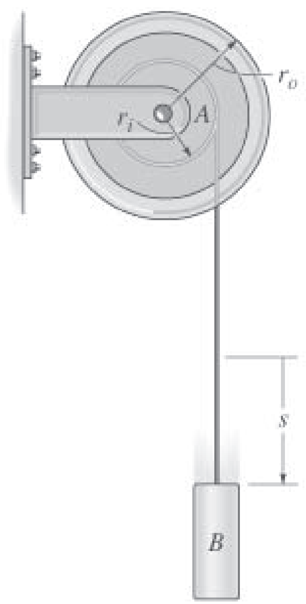
$$\mu_C N_C r = -M k_B^{\,2} \alpha$$

$$\begin{pmatrix} F_{AB} \\ N_C \\ \alpha \end{pmatrix} = \text{Find}(F_{AB}, N_C, \alpha) \qquad \begin{pmatrix} F_{AB} \\ N_C \end{pmatrix} = \begin{pmatrix} 111.477 \\ 178.364 \end{pmatrix} \text{N} \qquad \alpha = -31.709\ \frac{\text{rad}}{\text{s}^2}$$

$$t = \frac{\omega_1}{-\alpha} \qquad t = 1.261\ \text{s}$$

$$\mathbf{F_A} = \frac{F_{AB}}{\sqrt{a^2 + b^2}} \begin{pmatrix} a \\ b \end{pmatrix} \qquad \mathbf{F_A} = \begin{pmatrix} 89.2 \\ 66.9 \end{pmatrix} \text{N}$$

Problem 17-80



The cord is wrapped around the inner core of the spool. If block B of weight W_B is suspended from the cord and released from rest, determine the spool's angular velocity when $t = t_1$. Neglect the mass of the cord. The spool has weight W_S and the radius of gyration about the axle A is k_A. Solve the problem in two ways, first by considering the "system" consisting of the block and spool, and then by considering the block and spool separately.

Given:

$W_B = 5\ \text{lb}$

$t_1 = 3\ \text{s}$

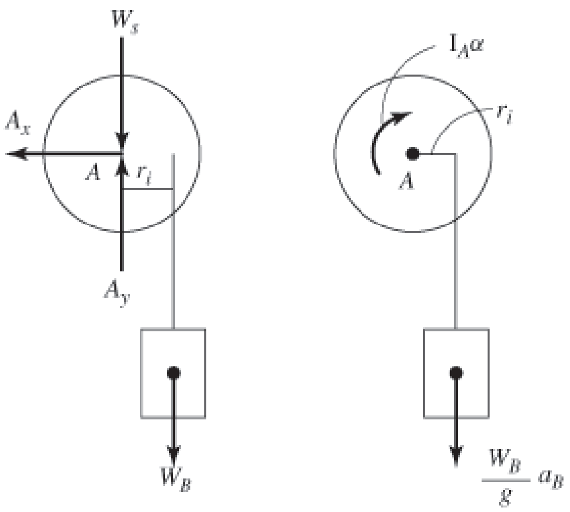
$W_S = 180\ \text{lb}$

$k_A = 1.25\ \text{ft}$

$r_i = 1.5\ \text{ft}$

$r_o = 2.75\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution:

(a) System as a whole

$$W_B r_i = \left(\frac{W_S}{g}\right)k_A{}^2\alpha + \left(\frac{W_B}{g}\right)(r_i\alpha r_i) \qquad \alpha = \frac{W_B r_i g}{W_B r_i{}^2 + W_S k_A{}^2} \qquad \alpha = 0.826\,\frac{\text{rad}}{\text{s}^2}$$

$$\omega = \alpha t_1 \qquad \omega = 2.477\,\frac{\text{rad}}{\text{s}}$$

(b) Parts separately Guesses $T = 1\ \text{lb}$ $\alpha = 1\,\frac{\text{rad}}{\text{s}^2}$

Given $T r_i = \left(\frac{W_S}{g}\right)k_A{}^2\alpha$ $\quad T - W_B = \left(\frac{-W_B}{g}\right)\alpha r_i$ $\quad \begin{pmatrix} T \\ \alpha \end{pmatrix} = \text{Find}(T, \alpha)$

$T = 4.808\,\text{lb}$ $\quad \alpha = 0.826\,\frac{\text{rad}}{\text{s}^2}$ $\quad \omega = \alpha t_1$ $\quad \omega = 2.477\,\frac{\text{rad}}{\text{s}}$

Problem 17-81

A boy of mass m_b sits on top of the large wheel which has mass m_w and a radius of gyration k_G. If the boy essentially starts from rest at $\theta = 0°$, and the wheel begins to rotate freely, determine the angle at which the boy begins to slip. The coefficient of static friction between the wheel and the boy is μ_s. Neglect the size of the boy in the calculation.

Given:

$m_b = 40\ \text{kg}$

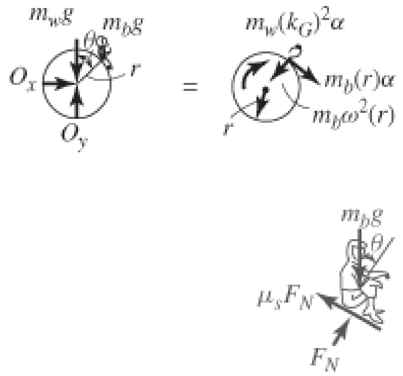
$m_w = 400\ \text{kg}$

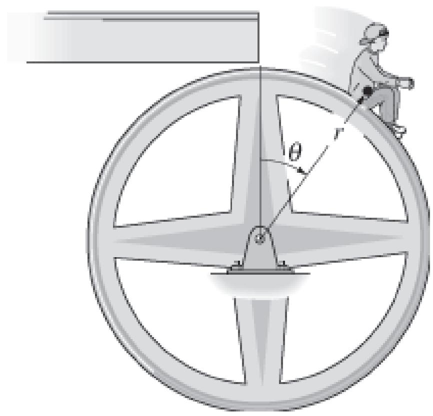
$k_G = 5.5\ \text{m}$

$\mu_s = 0.5$

$r = 8\ \text{m}$

$g = 9.81\,\frac{\text{m}}{\text{s}^2}$





Solution: Assume slipping occurs before contact is lost

$$m_b g r \sin(\theta) = \left(m_b r^2 + m_w k_G{}^2\right)\alpha \qquad \alpha = \frac{m_b g r}{m_b r^2 + m_w k_G{}^2}\sin(\theta)$$

Guesses $\theta = 10\ \text{deg}$ $\quad \alpha = 1\,\frac{\text{rad}}{\text{s}^2}$ $\quad \omega = 1\,\frac{\text{rad}}{\text{s}}$ $\quad F_N = 1\ \text{N}$

Given $\quad F_N - m_b g\cos(\theta) = -m_b r\omega^2 \qquad \mu_s F_N - m_b g\sin(\theta) = -m_b r\alpha$

$$\alpha = \frac{m_b g r}{m_b r^2 + m_w k_G^2}\sin(\theta) \qquad \frac{\omega^2}{2} = \frac{m_b g r}{m_b r^2 + m_w k_G^2}(1 - \cos(\theta))$$

$$\begin{pmatrix} \theta \\ \alpha \\ \omega \\ F_N \end{pmatrix} = \text{Find}(\theta, \alpha, \omega, F_N)$$

Since $F_N = 322\ \text{N}$ > 0 our assumption is correct.

$\alpha = 0.107\ \frac{\text{rad}}{\text{s}^2} \qquad \omega = 0.238\ \frac{\text{rad}}{\text{s}} \qquad \theta = 29.8\ \text{deg}$

Problem 17-82

The "Catherine wheel" is a firework that consists of a coiled tube of powder which is pinned at its center. If the powder burns at a constant rate m' such that the exhaust gases always exert a force having a constant magnitude of F, directed tangent to the wheel, determine the angular velocity of the wheel when k of the mass is burned off. Initially, the wheel is at rest and has mass m_0 and radius r_0. For the calculation, consider the wheel to always be a thin disk.

Given:

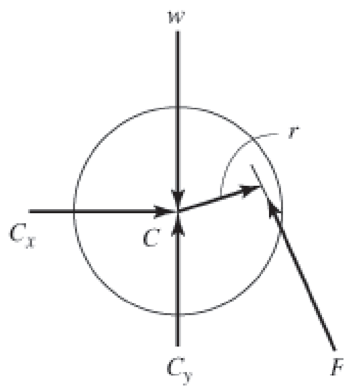
$m' = 20\ \frac{\text{gm}}{\text{s}}$

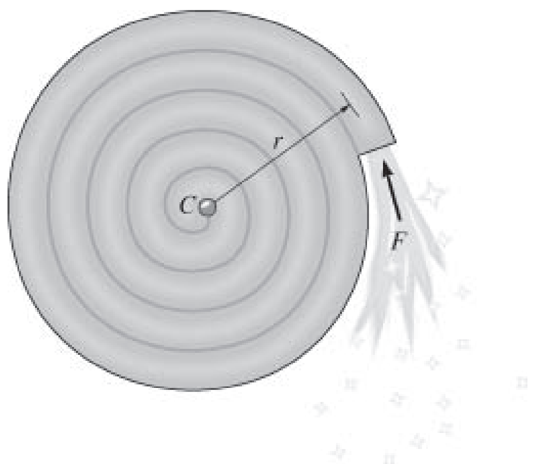
$F = 0.3\ \text{N}$

$m_0 = 100\ \text{gm}$

$r_0 = 75\ \text{mm}$

$k = 0.75$





Solution:

The density is $\quad \rho = \frac{m_0}{\pi r_0^2}$

The mass is $\quad m = m_0 - m' t = (1 - k) m_0$

$$t_1 = \frac{k m_0}{m'} \qquad t_1 = 3.75\ \text{s}$$

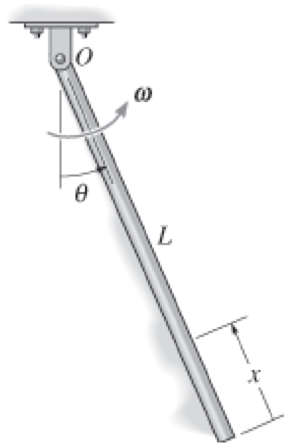
Find the radius $\quad m_0 - m' t = \rho\, \pi r^2 \qquad r = \sqrt{\frac{m_0 - m' t}{\rho\, \pi}}$

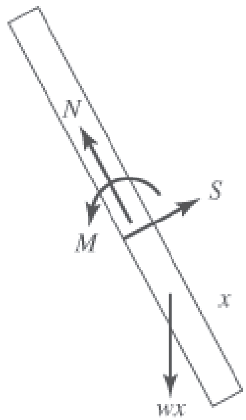
Dynamics $\quad F r = \frac{1}{2} m r^2 \alpha \qquad \alpha = \frac{2F\sqrt{\rho \pi}}{\sqrt{(m_0 - m' t)^3}}$

$$\omega = \int_0^{t_1} \frac{2F\sqrt{\rho \pi}}{\sqrt{(m_0 - m' t)^3}}\, dt \qquad \omega = 800\,\frac{\text{rad}}{\text{s}}$$

Problem 17-83

The bar has a weight per length of w. If it is rotating in the vertical plane at a constant rate ω about point O, determine the internal normal force, shear force, and moment as a function of x and θ.





Solution:

$$N - w x \cos(\theta) = \left(\frac{w x}{g}\right)\omega^2\left(L - \frac{x}{2}\right) \qquad N = w x\left[\cos(\theta) + \left(\frac{\omega^2}{g}\right)\left(L - \frac{x}{2}\right)\right]$$

$$S - w x \sin(\theta) = 0 \qquad S = w x \,\text{win}(\theta)$$

$$M - S\frac{x}{2} = 0 \qquad M = w\frac{x^2}{2}\sin(\theta)$$

Problem 17-84

A force F is applied perpendicular to the axis of the rod of weight W and moves from O to A at a constant rate v. If the rod is at rest when $\theta = 0°$ and F is at O when $t = 0$, determine the rod's angular velocity at the instant the force is at A. Through what angle has the rod rotated when this occurs? The rod rotates in the *horizontal plane*.

Given:

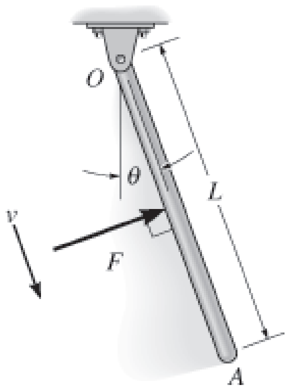
$F = 2$ lb

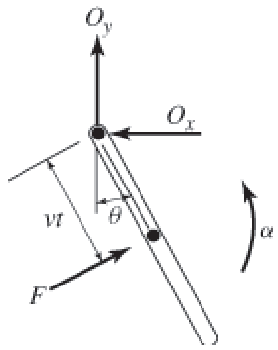
$W = 5 \text{ lb}$

$v = 4 \frac{\text{ft}}{\text{s}}$

$L = 4 \text{ ft}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$





Solution:

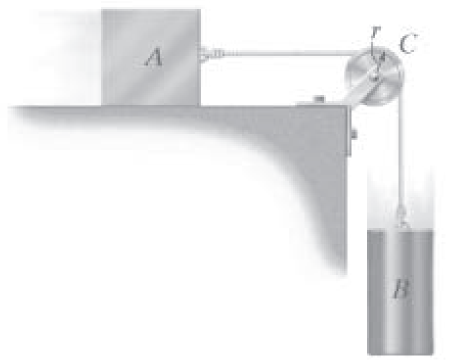
$F v t = \frac{1}{3} \frac{W}{g} L^2 \alpha$ $\quad t = \frac{L}{v}$ $\quad t = 1 \text{ s}$

$\alpha = \left(\frac{3F v g}{W L^2}\right) t$ $\quad \omega = \left(\frac{3F v g}{2W L^2}\right) t^2$ $\quad \omega = 4.83 \frac{\text{rad}}{\text{s}}$

$\theta = \left(\frac{F v g}{2W L^2}\right) t^3$ $\quad \theta = 92.2 \text{ deg}$

Problem 17-85

Block A has a mass m and rests on a surface having a coefficient of kinetic friction μ_k. The cord attached to A passes over a pulley at C and is attached to a block B having a mass $2m$. If B is released, determine the acceleration of A. Assume that the cord does not slip over the pulley. The pulley can be approximated as a thin disk of radius r and mass $m/4$. Neglect the mass of the cord.



Solution:

Given

$T_1 - \mu_k m g = m a$

$T_2 - 2m g = -2m a$

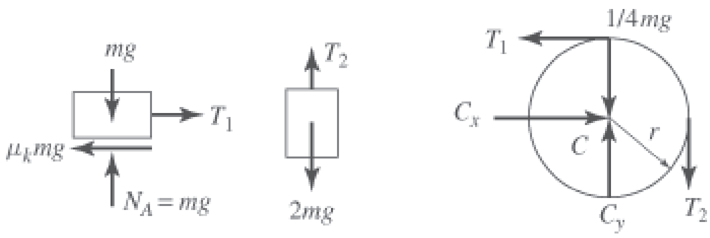
$T_1 r - T_2 r = -\frac{1}{2}\left(\frac{m}{4}\right) r^2 \alpha$

$a = \alpha r$

Solving

$T_1 = \frac{m g}{25}\left(16 + 17\mu_k\right)$

$T_2 = \frac{2m g}{25}\left(9 + 8\mu_k\right)$



$$\alpha = \frac{8g}{25r}(2 - \mu_k)$$

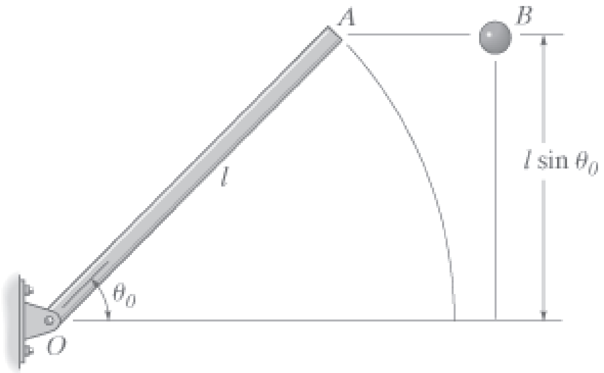
$$a = \frac{8g}{25}(2 - \mu_k)$$

Problem 17-86

The slender rod of mass m is released from rest when $\theta = \theta_0$. At the same instant ball B having the same mass m is released. Will B or the end A of the rod have the greatest speed when they pass the horizontal? What is the difference in their speeds?

Given:

$$\theta_0 = 45 \text{ deg}$$



Solution: At horizontal $\theta_f = 0 \text{ deg}$

Rod

$$m g \frac{1}{2}\cos(\theta) = \frac{1}{3} m l^2 \alpha$$

$$\alpha = \frac{3g}{2l}\cos(\theta)$$

$$\frac{\omega^2}{2} = \frac{3g}{2l}(\sin(\theta_0) - \sin(\theta_f))$$

$$\omega = \sqrt{\frac{3g}{l}(\sin(\theta_0) - \sin(\theta_f))} \qquad v_A = \omega l = \sqrt{3 g l(\sin(\theta_0) - \sin(\theta_f))}$$

Ball

$$m g = m a \qquad a = g$$

$$\frac{v^2}{2} = g l(\sin(\theta_0) - \sin(\theta_f))$$

$$v_B = \sqrt{2 g l(\sin(\theta_0) - \sin(\theta_f))}$$

Define the constant $k = (\sqrt{3} - \sqrt{2})\sqrt{\sin(\theta_0) - \sin(\theta_f)}$

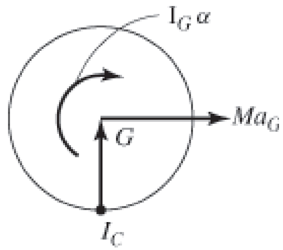
A has the greater speed and the difference is given by $\Delta v = k\sqrt{g l}$

$$k = 0.267$$

Problem 17-87

If a disk *rolls without slipping*, show that when moments are summed about the instantaneous center of zero velocity, *IC*, it is possible to use the moment equation $\Sigma M_{IC} = I_{IC}\alpha$, where I_{IC} represents the moment of inertia of the disk calculated about the instantaneous axis of zero velocity.

Solution:



$\Sigma M_{IC} = \Sigma(M_k)_{IC}$; $\quad \Sigma M_{IC} = I_G\alpha + ma_G r$

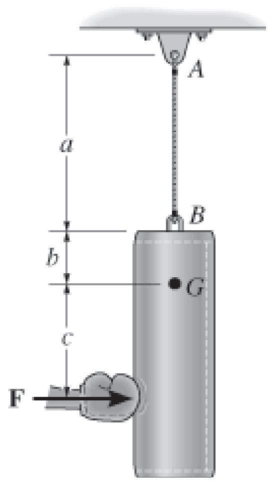
Since there is no slipping, $a_G = \alpha r$

Thus, $\quad \Sigma M_{IC} = \left(I_G + m r^2\right)\alpha$

By the parallel - axis theorem, the term in parenthesis represents I_{IC}.

$$\Sigma M_{IC} = I_{IC}\alpha \qquad \text{Q.E.D}$$

***Problem 17-88**



The punching bag of mass M has a radius of gyration about its center of mass G of k_G. If it is subjected to a horizontal force F, determine the initial angular acceleration of the bag and the tension in the supporting cable AB.

Given:

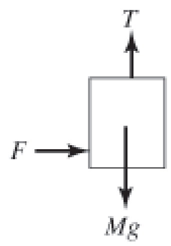
$M = 20 \text{ kg}$ $\quad b = 0.3 \text{ m}$

$k_G = 0.4 \text{ m}$ $\quad c = 0.6 \text{ m}$

$F = 30 \text{ N}$ $\quad g = 9.81 \frac{\text{m}}{\text{s}^2}$

$a = 1 \text{ m}$

Solution:



$T - Mg = 0 \qquad T = Mg \qquad T = 196.2 \text{ N}$

$F c = M k_G^2 \alpha \qquad \alpha = \frac{F c}{M k_G^2} \qquad \alpha = 5.625 \frac{\text{rad}}{\text{s}^2}$

Problem 17-89

The trailer has mass M_1 and a mass center at G, whereas the spool has mass M_2, mass center at O, and radius of gyration about an axis passing through O k_O. If a force F is applied to the cable, determine the angular acceleration of the spool and the acceleration of the trailer. The wheels have negligible mass and are free to roll.

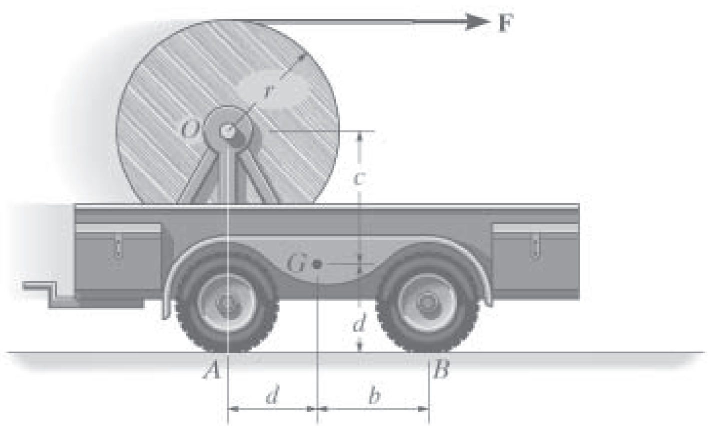
Given:

$M_1 = 580\ \text{kg}$ $\quad b = 0.5\ \text{m}$

$M_2 = 200\ \text{kg}$ $\quad c = 0.6\ \text{m}$

$k_O = 0.45\ \text{m}$ $\quad d = 0.4\ \text{m}$

$F = 60\ \text{N}$ $\quad r = 0.5\ \text{m}$

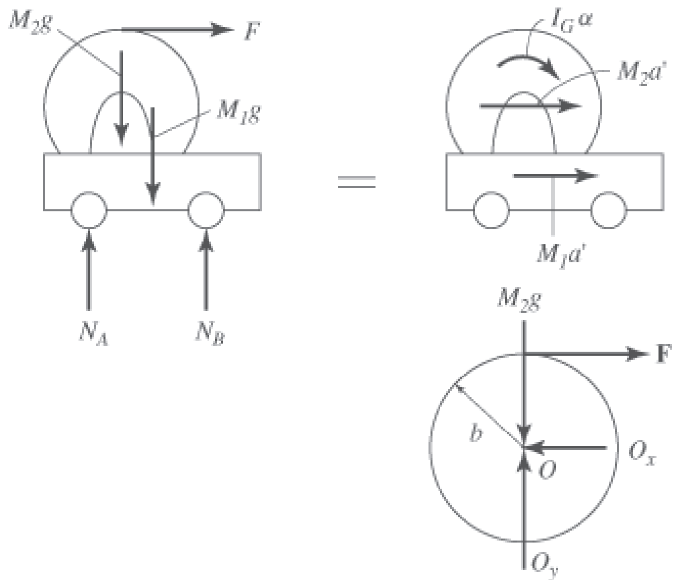


Solution:

$$F = (M_1 + M_2)a$$

$$a = \frac{F}{M_1 + M_2}$$

$$a = 0.0769\ \frac{\text{m}}{\text{s}^2}$$



$$F r = M_2 k_O^2 \alpha$$

$$\alpha = F\left(\frac{r}{M_2 k_O^2}\right)$$

$$\alpha = 0.741\ \frac{\text{rad}}{\text{s}^2}$$

Problem 17-90

The rocket has weight W, mass center at G, and radius of gyration about the mass center k_G when it is fired. Each of its two engines provides a thrust T. At a given instant, engine A suddenly fails to operate. Determine the angular acceleration of the rocket and the acceleration of its nose B.

Given:

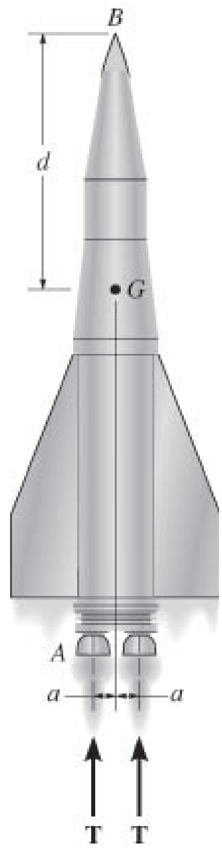
$W = 20000\ \text{lb}$ $T = 50000\ \text{lb}$

$k_G = 21\ \text{ft}$ $\quad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$d = 30\ \text{ft}$

$a = 1.5\ \text{ft}$

Solution:



$$T a = \left(\frac{W}{g}\right) k_G^{\,2} \alpha$$

$$\alpha = \frac{T a g}{W k_G^{\,2}} \qquad \alpha = 0.274\ \frac{\text{rad}}{\text{s}^2}$$

$$T - W = \left(\frac{W}{g}\right) a_{Gy}$$

$$a_{Gy} = \frac{(T - W)g}{W} \qquad a_{Gy} = 14.715\ \frac{\text{m}}{\text{s}^2}$$

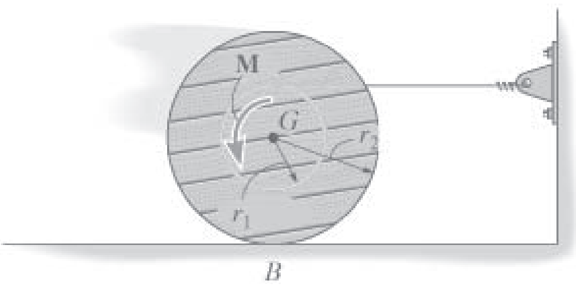
$$\mathbf{a_B} = \begin{pmatrix} 0 \\ a_{Gy} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} 0 \\ d \\ 0 \end{pmatrix} \qquad \mathbf{a_B} = \begin{pmatrix} -8.2 \\ 48.3 \\ 0.0 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

$$\left|\mathbf{a_B}\right| = 49.0\ \frac{\text{ft}}{\text{s}^2}$$

$$\theta = \operatorname{atan}\left(\frac{a_{Gy}}{\alpha d}\right) \qquad \theta = 80.3\ \text{deg}$$

Problem 17-91

The spool and wire wrapped around its core have a mass m_s and a centroidal radius of gyration k_G. If the coefficient of kinetic friction at the ground is μ_k, determine the angular acceleration of the spool when the couple M is applied.



Given:

$m_s = 20\ \text{kg}$ $\quad M = 30\ \text{N m}$

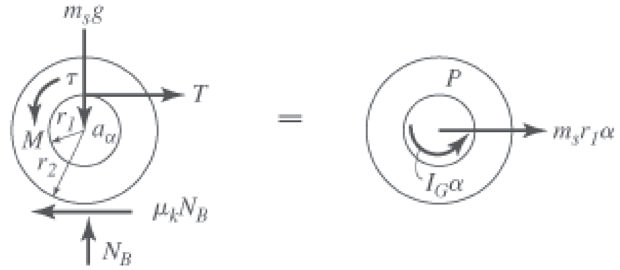
$k_G = 250\ \text{mm}$ $\quad r_1 = 200\ \text{mm}$

$\mu_k = 0.1$ $\quad r_2 = 400\ \text{mm}$

Solution:

Guesses $T = 1\ \text{N}$ $N_B = 1\ \text{N}$

$\alpha = 1\ \dfrac{\text{rad}}{\text{s}^2}$



Given

$$T - \mu_k N_B = m_s r_1 \alpha$$

$$N_B - m_s g = 0$$

$$M - \mu_k N_B r_2 - T r_1 = m_s k_G^{\,2} \alpha$$

$$\begin{pmatrix} T \\ N_B \\ \alpha \end{pmatrix} = \text{Find}(T, N_B, \alpha) \qquad \begin{pmatrix} T \\ N_B \end{pmatrix} = \begin{pmatrix} 55.2 \\ 196.2 \end{pmatrix} \text{N} \qquad \alpha = 8.89\ \frac{\text{rad}}{\text{s}^2}$$

***Problem 17-92**

The uniform board of weight W is suspended from cords at C and D. If these cords are subjected to constant forces F_A and F_B respectively, determine the acceleration of the board's center and the board's angular acceleration. Assume the board is a thin plate. Neglect the mass of the pulleys at E and F.

Given:

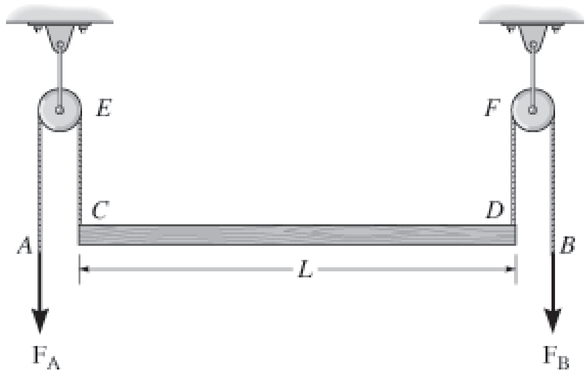
$W = 50\ \text{lb}$

$F_A = 30\ \text{lb}$

$F_B = 45\ \text{lb}$

$L = 10\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution:

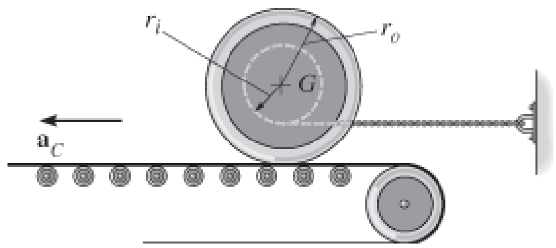
$$F_A + F_B - W = \left(\frac{W}{g}\right) a_{Gy} \qquad a_{Gy} = \left(\frac{F_A + F_B - W}{W}\right) g \qquad a_{Gy} = 16.1\ \frac{\text{ft}}{\text{s}^2}$$

$$F_B\left(\frac{L}{2}\right) - F_A\left(\frac{L}{2}\right) = \frac{1}{12}\left(\frac{W}{g}\right)L^2\alpha \qquad \alpha = \frac{6(F_B - F_A)g}{WL} \qquad \alpha = 5.796\,\frac{\text{rad}}{\text{s}^2}$$

Problem 17-93

The spool has mass M and radius of gyration k_G. It rests on the surface of a conveyor belt for which the coefficient of static friction is μ_s and the coefficient of kinetic friction is μ_k. If the conveyor accelerates at rate a_C, determine the initial tension in the wire and the angular acceleration of the spool. The spool is originally at rest.

Units Used: $\quad \text{kN} = 10^3\ \text{N}$



Given:

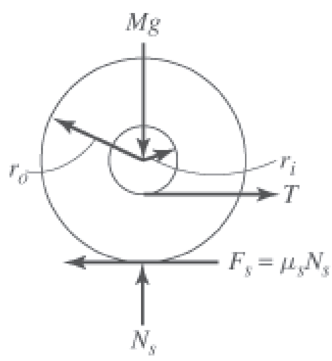
$M = 500\ \text{kg} \qquad a_C = 1\,\frac{\text{m}}{\text{s}^2}$

$k_G = 1.30\ \text{m} \qquad r_i = 0.8\ \text{m}$

$\mu_s = 0.5 \qquad r_o = 1.6\ \text{m}$

$\mu_k = 0.4$

Solution: $\quad$ Assume no slip



Guesses $\quad \alpha = 1\,\frac{\text{rad}}{\text{s}^2} \qquad a_x = 1\,\frac{\text{m}}{\text{s}^2} \qquad T = 1\text{N}$

$F_{max} = 1\text{N} \qquad N_s = 1\text{N} \qquad F_s = 1\text{N}$

Given $\quad T - F_s = M a_x \qquad N_s - M g = 0$

$T r_i - F_s r_o = -M k_G^2 \alpha \qquad F_{max} = \mu_s N_s$

$a_x = r_i\alpha \qquad a_C = (r_o - r_i)\alpha$

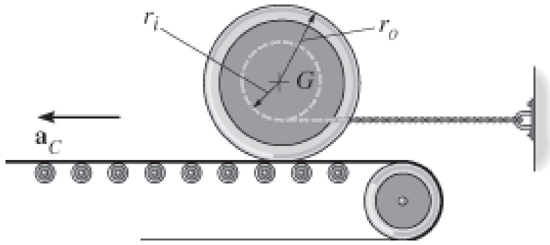
$$\begin{pmatrix} \alpha \\ a_x \\ F_{max} \\ N_s \\ F_s \\ T \end{pmatrix} = \text{Find}(\alpha, a_x, F_{max}, N_s, F_s, T) \qquad \begin{pmatrix} N_s \\ F_s \\ F_{max} \end{pmatrix} = \begin{pmatrix} 4.907 \\ 1.82 \\ 2.454 \end{pmatrix}\text{kN} \qquad \alpha = 1.25\,\frac{\text{rad}}{\text{s}^2}$$

$$T = 2.32\,\text{kN}$$

Since $F_s = 1.82\,\text{kN} < F_{max} = 2.454\,\text{kN}$ then our no-slip assumption is correct.

Problem 17-94

The spool has mass M and radius of gyration k_G. It rests on the surface of a conveyor belt for which the coefficient of static friction is μ_s. Determine the greatest acceleration of the conveyor so that the spool will not slip. Also, what are the initial tension in the wire and the angular acceleration of the spool? The spool is originally at rest.



Units Used: $\quad kN = 10^3\ N$

Given: $\quad M = 500\ kg$

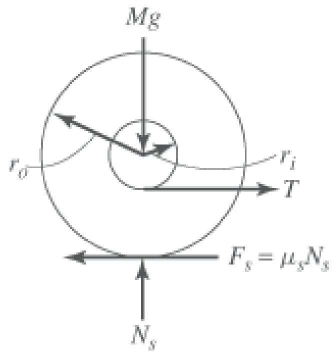
$k_G = 1.30\ m \quad r_i = 0.8\ m$

$\mu_s = 0.5 \quad r_o = 1.6\ m$

Solution:

Guesses $\quad \alpha = 1\ \frac{rad}{s^2} \quad a_x = 1\ \frac{m}{s^2} \quad a_C = 1\ \frac{m}{s^2}$

$N_s = 1\ N \quad F_s = 1\ N \quad T = 1\ N$



Given $\quad T - F_s = M a_x \qquad N_s - M g = 0$

$T r_i - F_s r_o = -M k_G^2 \alpha \qquad F_s = \mu_s N_s$

$a_x = r_i \alpha \qquad a_C = (r_o - r_i)\alpha$

$$\begin{pmatrix} \alpha \\ a_x \\ a_C \\ N_s \\ F_s \\ T \end{pmatrix} = \text{Find}(\alpha, a_x, a_C, N_s, F_s, T) \qquad \begin{pmatrix} N_s \\ F_s \end{pmatrix} = \begin{pmatrix} 4.907 \\ 2.454 \end{pmatrix} kN \qquad a_C = 1.348\ \frac{m}{s^2}$$

$$\alpha = 1.685\ \frac{rad}{s^2} \qquad T = 3.128\ kN$$

Problem 17-95

The wheel has weight W and radius of gyration k_G. If the coefficients of static and kinetic friction between the wheel and the plane are μ_s and μ_k, determine the wheel's angular acceleration as it rolls down the incline.

Given:

$W = 30\ \text{lb}$

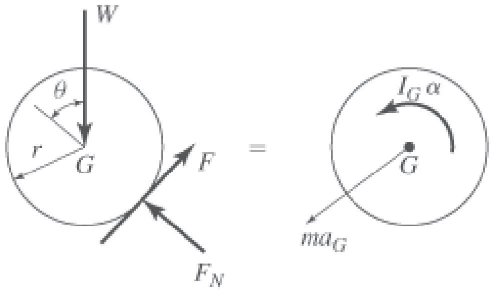
$k_G = 0.6\ \text{ft}$

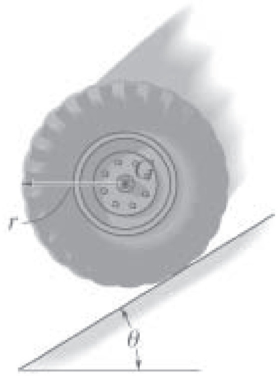
$\mu_s = 0.2$

$\mu_k = 0.15$

$r = 1.25\ \text{ft}$

$\theta = 12\ \text{deg}$





Solution: Assume no slipping

Guesses $F_N = 1\ \text{lb}$ $F = 1\ \text{lb}$ $a_G = 1\ \frac{\text{ft}}{\text{s}^2}$ $\alpha = 1\ \frac{\text{rad}}{\text{s}^2}$ $F_{max} = 1\ \text{lb}$

Given $F - W\sin(\theta) = \frac{-W}{g}a_G$ $F_N - W\cos(\theta) = 0$ $F_{max} = \mu_s F_N$

$F\,r = \frac{W}{g}k_G^{\,2}\alpha$ $a_G = r\alpha$

$$\begin{pmatrix} F \\ F_N \\ F_{max} \\ a_G \\ \alpha \end{pmatrix} = \text{Find}(F, F_N, F_{max}, a_G, \alpha) \qquad \begin{pmatrix} F \\ F_N \\ F_{max} \end{pmatrix} = \begin{pmatrix} 1.17 \\ 29.34 \\ 5.87 \end{pmatrix}\text{lb} \qquad a_G = 5.44\ \frac{\text{ft}}{\text{s}^2}$$

$$\alpha = 4.35\ \frac{\text{rad}}{\text{s}^2}$$

Since $F = 1.17\ \text{lb} < F_{max} = 5.87\ \text{lb}$ then our no-slip assumption is correct.

***Problem 17-96**

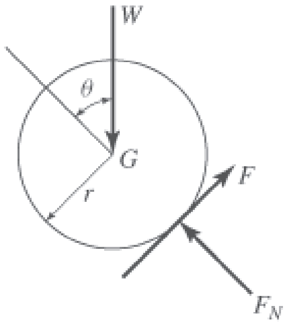
The wheel has a weight W and a radius of gyration k_G. If the coefficients of static and kinetic friction between the wheel and the plane are μ_s and μ_k, determine the maximum angle θ of the inclined plane so that the wheel rolls without slipping.

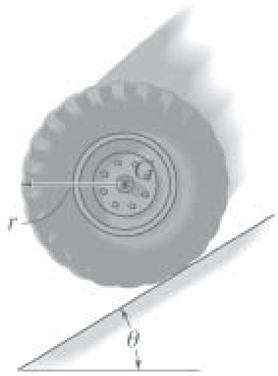
Given:

$W = 30\ \text{lb}$ $\quad r = 1.25\ \text{ft}$

$k_G = 0.6\ \text{ft}$ $\quad \theta = 12\ \text{deg}$

$\mu_s = 0.2$ $\quad \mu_k = 0.15$





Solution:

Guesses

$\theta = 1\ \text{deg}$ $\quad F_N = 1\ \text{lb}$ $\quad F = 1\ \text{lb}$

$a_G = 1\ \dfrac{\text{ft}}{\text{s}^2}$ $\quad \alpha = 1\ \dfrac{\text{rad}}{\text{s}^2}$

Given

$$F - W\sin(\theta) = \left(\frac{-W}{g}\right)a_G \qquad F\,r = \left(\frac{W}{g}\right)k_G^{\,2}\alpha$$

$$F_N - W\cos(\theta) = 0 \qquad F = \mu_s F_N$$

$$a_G = r\alpha$$

$$\begin{pmatrix}\theta \\ F_N \\ F \\ a_G \\ \alpha\end{pmatrix} = \text{Find}(\theta, F_N, F, a_G, \alpha) \qquad \begin{pmatrix}F \\ F_N\end{pmatrix} = \begin{pmatrix}4.10 \\ 20.50\end{pmatrix}\text{lb} \qquad a_G = 19.1\,\frac{\text{ft}}{\text{s}^2} \qquad \alpha = 15.3\,\frac{\text{rad}}{\text{s}^2}$$

$$\theta = 46.9\ \text{deg}$$

Problem 17-97

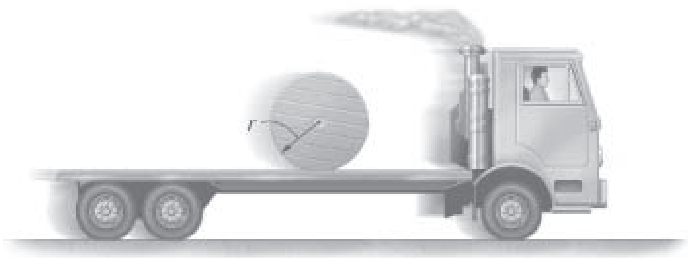
The truck carries the spool which has weight W and radius of gyration k_G. Determine the angular acceleration of the spool if it is not tied down on the truck and the truck begins to accelerate at the rate a_{At}. Assume the spool does not slip on the bed of the truck.

Given:

$W = 500\ \text{lb}$ $\quad r = 3\ \text{ft}$

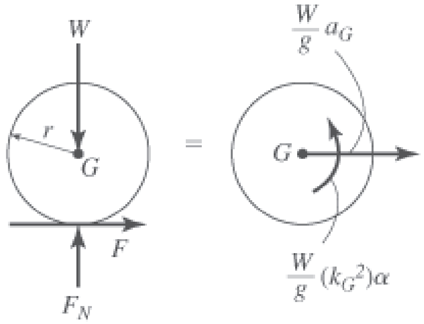
$k_G = 2\ \text{ft}$ $\quad g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

$a_{At} = 3\ \dfrac{\text{ft}}{\text{s}^2}$



Solution:

Guesses $a_G = 1\ \frac{\text{ft}}{\text{s}^2}$ $\alpha = 1\ \frac{\text{rad}}{\text{s}^2}$



Given

$$a_G = a_{At} - \alpha r$$

$$0 = \left(\frac{-W}{g}\right)a_G r + \left(\frac{W}{g}\right)k_G^2\,\alpha$$

$$\begin{pmatrix} a_G \\ \alpha \end{pmatrix} = \text{Find}(a_G, \alpha) \qquad a_G = 0.923\ \frac{\text{ft}}{\text{s}^2} \qquad \alpha = 0.692\ \frac{\text{rad}}{\text{s}^2}$$

Problem 17-98

The truck carries the spool which has weight W and radius of gyration k_G. Determine the angular acceleration of the spool if it is not tied down on the truck and the truck begins to accelerate at the rate a_{At}. The coefficients of static and kinetic friction between the spool and the truck bed are μ_s and μ_k, respectively.

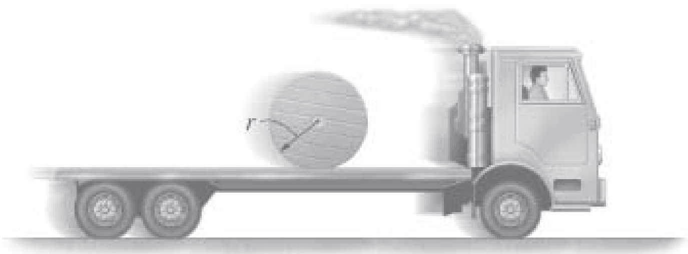
Given:

$W = 200\ \text{lb}$ $g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$k_G = 2\ \text{ft}$ $\mu_s = 0.15$

$a_{At} = 5\ \frac{\text{ft}}{\text{s}^2}$ $\mu_k = 0.1$

$r = 3\ \text{ft}$

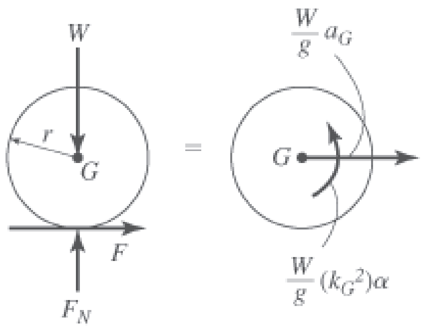


Solution: Assume no slip

Guesses

$F = 1\ \text{lb}$ $F_N = 1\ \text{lb}$ $F_{max} = 1\ \text{lb}$

$a_G = 1\ \frac{\text{ft}}{\text{s}^2}$ $\alpha = 1\ \frac{\text{rad}}{\text{s}^2}$



Given $F = \frac{W}{g}a_G$ $F\,r = \frac{W}{g}k_G^2\,\alpha$ $F_N - W = 0$ $F_{max} = \mu_s F_N$

$$a_G = a_{At} - \alpha r$$

$$\begin{pmatrix} F \\ F_N \\ F_{max} \\ a_G \\ \alpha \end{pmatrix} = \text{Find}(F, F_N, F_{max}, a_G, \alpha) \qquad \begin{pmatrix} F \\ F_{max} \\ F_N \end{pmatrix} = \begin{pmatrix} 9.56 \\ 30.00 \\ 200.00 \end{pmatrix} \text{lb} \qquad a_G = 1.538 \frac{\text{ft}}{\text{s}^2}$$

$$\alpha = 1.154 \frac{\text{rad}}{\text{s}^2}$$

Since $F = 9.56\,\text{lb} < F_{max} = 30\,\text{lb}$ then our no-slip assumption is correct.

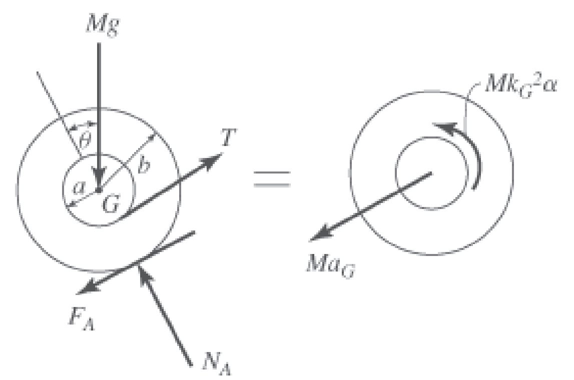
Problem 17-99

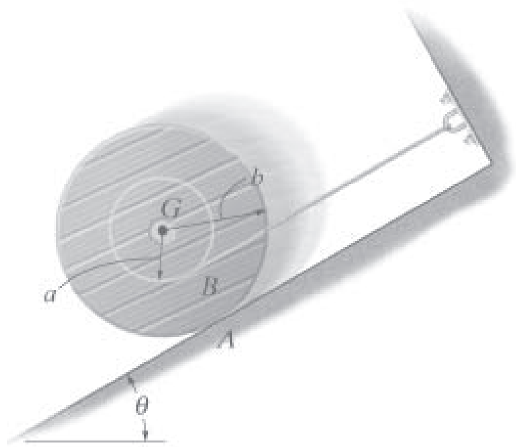
The spool has mass M and radius of gyration k_G. It rests on the inclined surface for which the coefficient of kinetic friction is μ_k. If the spool is released from rest and slips at A, determine the initial tension in the cord and the angular acceleration of the spool.

Given:

$M = 75\ \text{kg} \qquad k_G = 0.380\ \text{m} \qquad a = 0.3\ \text{m}$

$\mu_k = 0.15 \qquad \theta = 30\ \text{deg} \qquad b = 0.6\ \text{m}$





Solution:

Guesses $\qquad T = 1\ \text{N} \qquad N_A = 1\ \text{N} \qquad a_G = 1\ \frac{\text{m}}{\text{s}^2} \qquad \alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

Given

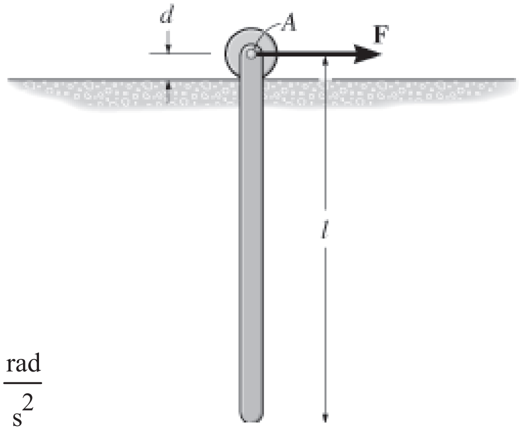
$$T - Mg\sin(\theta) - \mu_k N_A = -Ma_G \qquad N_A - Mg\cos(\theta) = 0$$

$$Ta - \mu_k N_A b = Mk_G^2\alpha \qquad a_G = \alpha a$$

$$\begin{pmatrix} T \\ N_A \\ \alpha \\ a_G \end{pmatrix} = \text{Find}(T, N_A, \alpha, a_G) \qquad a_G = 1.395\ \frac{\text{m}}{\text{s}^2} \qquad \alpha = 4.65\ \frac{\text{rad}}{\text{s}^2} \qquad T = 359\ \text{N}$$

***Problem 17-100**

A uniform rod having weight W is pin-supported at A from a roller which rides on horizontal track. If the rod is originally at rest, and horizontal force **F** is applied to the roller, determine the acceleration of the roller. Neglect the mass of the roller and its size d in the computations.



Given:

$W = 10\ \text{lb}$

$F = 15\ \text{lb}$

$l = 2\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

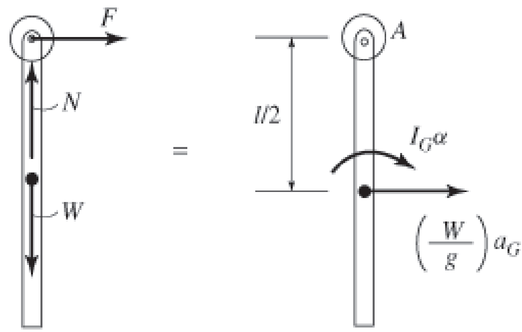
Guesses $\quad a_G = 1\ \frac{\text{ft}}{\text{s}^2} \quad a_A = 1\ \frac{\text{ft}}{\text{s}^2} \quad \alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

Given

$$F = \left(\frac{W}{g}\right)a_G \qquad F\left(\frac{l}{2}\right) = \frac{1}{12}\left(\frac{W}{g}\right)l^2\alpha$$

$$a_A = a_G + \alpha\frac{l}{2}$$

$$\begin{pmatrix} a_G \\ a_A \\ \alpha \end{pmatrix} = \text{Find}(a_G, a_A, \alpha)$$



$$a_G = 48.3\ \frac{\text{ft}}{\text{s}^2} \qquad \alpha = 145\ \frac{\text{rad}}{\text{s}^2} \qquad a_A = 193\ \frac{\text{ft}}{\text{s}^2}$$

Problem 17-101

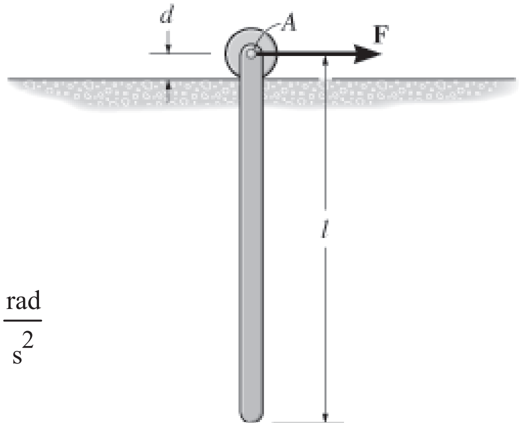
A uniform rod having weight W is pin-supported at A from a roller which rides on horizontal track. Assume that the roller at A is replaced by a slider block having a negligible mass. If the rod is initially at rest, and a horizontal force **F** is applied to the slider, determine the slider's acceleration. The coefficient of kinetic friction between the block and the track is μ_k. Neglect the dimension d and the size of the block in the computations.

Given:

$W = 10 \text{ lb}$ $\mu_k = 0.2$

$F = 15 \text{ lb}$ $g = 32.2 \frac{\text{ft}}{\text{s}^2}$

$l = 2 \text{ ft}$



Solution:

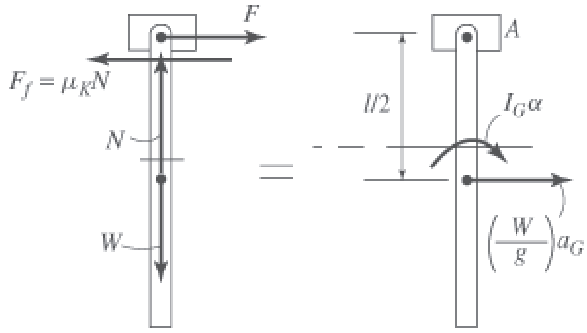
Guesses $a_G = 1 \frac{\text{ft}}{\text{s}^2}$ $a_A = 1 \frac{\text{ft}}{\text{s}^2}$ $\alpha = 1 \frac{\text{rad}}{\text{s}^2}$

Given

$$F - \mu_k W = \left(\frac{W}{g}\right) a_G$$

$$\left(F - \mu_k W\right)\frac{l}{2} = \frac{1}{12}\left(\frac{W}{g}\right) l^2 \alpha$$

$$a_A = a_G + \alpha\left(\frac{l}{2}\right)$$



$$\begin{pmatrix} a_G \\ a_A \\ \alpha \end{pmatrix} = \text{Find}\left(a_G, a_A, \alpha\right)$$

$a_G = 41.86 \frac{\text{ft}}{\text{s}^2}$ $\alpha = 126 \frac{\text{rad}}{\text{s}^2}$ $a_A = 167 \frac{\text{ft}}{\text{s}^2}$

Problem 17-102

The lawn roller has mass M and radius of gyration k_G. If it is pushed forward with a force **F** when the handle is in the position shown, determine its angular acceleration. The coefficients of static and kinetic friction between the ground and the roller are μ_s and μ_k, respectively.

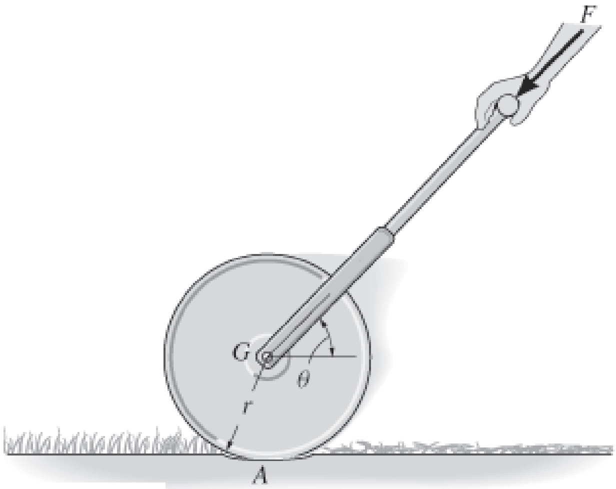
Given:

$M = 80 \text{ kg}$ $\mu_s = 0.12$

$k_G = 0.175 \text{ m}$ $\mu_k = 0.1$

$F = 200 \text{ N}$ $r = 200 \text{ mm}$

$\theta = 45 \text{ deg}$ $g = 9.81 \dfrac{\text{m}}{\text{s}^2}$



Solution: Assume no slipping,

Guesses

$F_f = 1 \text{ N}$ $F_N = 1 \text{ N}$ $a_x = 1 \dfrac{\text{m}}{\text{s}^2}$ $\alpha = 1 \dfrac{\text{rad}}{\text{s}^2}$ $F_{max} = 1 \text{ N}$

Given $-F\cos(\theta) + F_f = -M a_x$ $F_N - Mg - F\sin(\theta) = 0$

$F_f r = M k_G^2 \alpha$ $a_x = \alpha r$ $F_{max} = \mu_s F_N$

$$\begin{pmatrix} F_f \\ F_N \\ F_{max} \\ a_x \\ \alpha \end{pmatrix} = \text{Find}(F_f, F_N, F_{max}, a_x, \alpha) \qquad \begin{pmatrix} F_N \\ F_f \\ F_{max} \end{pmatrix} = \begin{pmatrix} 926.221 \\ 61.324 \\ 111.147 \end{pmatrix} \text{N} \qquad a_x = 1.001 \frac{\text{m}}{\text{s}^2}$$

$$\alpha = 5.01 \frac{\text{rad}}{\text{s}^2}$$

Since $F_f = 61.3 \text{ N} < F_{max} = 111.1 \text{ N}$ then our no-slip assumption is true.

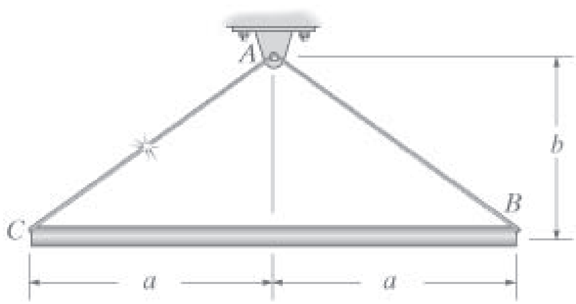
Problem 17-103

The slender bar of weight W is supported by two cords AB and AC. If cord AC suddenly breaks, determine the initial angular acceleration of the bar and the tension in cord AB.

Given:

$W = 150 \text{ lb} \qquad a = 4 \text{ ft}$

$g = 32.2 \dfrac{\text{ft}}{\text{s}^2} \qquad b = 3 \text{ ft}$

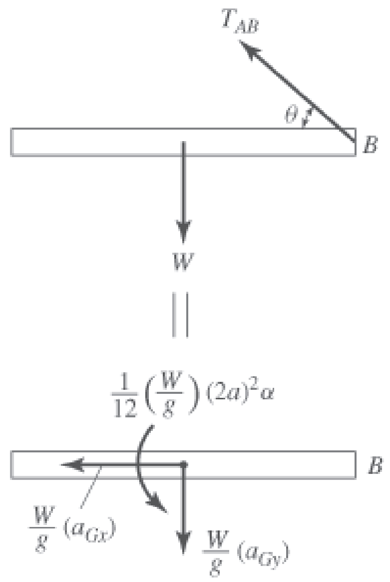


Solution: $\theta = \text{atan}\left(\dfrac{b}{a}\right)$

Guesses

$T_{AB} = 1 \text{ lb} \qquad \alpha = 1 \dfrac{\text{rad}}{\text{s}^2}$

$a_{Gx} = 1 \dfrac{\text{ft}}{\text{s}^2} \qquad a_{Gy} = 1 \dfrac{\text{ft}}{\text{s}^2} \qquad a_B = 1 \dfrac{\text{ft}}{\text{s}^2}$



Given

$$-T_{AB}\cos(\theta) = \left(\frac{-W}{g}\right)a_{Gx}$$

$$T_{AB}\sin(\theta) - W = \left(\frac{-W}{g}\right)a_{Gy} \qquad -a_{Gx} = -a_B\sin(\theta)$$

$$T_{AB}\sin(\theta)a = \frac{1}{12}\left(\frac{W}{g}\right)(2a)^2\alpha \qquad -a_{Gy} + \alpha a = -a_B\cos(\theta)$$

$$\begin{pmatrix} T_{AB} \\ \alpha \\ a_{Gx} \\ a_{Gy} \\ a_B \end{pmatrix} = \text{Find}(T_{AB}, \alpha, a_{Gx}, a_{Gy}, a_B) \qquad \begin{pmatrix} a_{Gx} \\ a_{Gy} \\ a_B \end{pmatrix} = \begin{pmatrix} 7.43 \\ 26.63 \\ 12.38 \end{pmatrix} \frac{\text{ft}}{\text{s}^2} \qquad \alpha = 4.18 \frac{\text{rad}}{\text{s}^2}$$

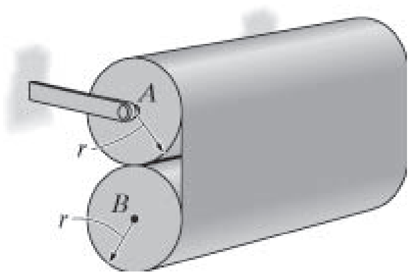
$$T_{AB} = 43.3 \text{ lb}$$

***Problem 17-104**

A long strip of paper is wrapped into two rolls, each having mass M. Roll A is pin-supported about its center whereas roll B is not centrally supported. If B is brought into contact with A and released from rest, determine the initial tension in the paper between the rolls and the angular acceleration of each roll. For the calculation, assume the rolls to be approximated by cylinders.

Given:

$M = 8 \text{ kg} \qquad r = 90 \text{ mm} \qquad g = 9.81 \dfrac{\text{m}}{\text{s}^2}$



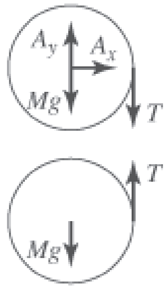
Solution: Guesses

$T = 1 \text{ N} \qquad a_{By} = 1 \dfrac{\text{m}}{\text{s}^2} \qquad \alpha_A = 1 \dfrac{\text{rad}}{\text{s}^2} \qquad \alpha_B = 1 \dfrac{\text{rad}}{\text{s}^2}$

Given

$$T r = \frac{1}{2} M r^2 \alpha_A \qquad T r = \frac{1}{2} M r^2 \alpha_B$$

$$T - M g = M a_{By} \qquad -\alpha_A r = a_{By} + \alpha_B r$$



$$\begin{pmatrix} T \\ a_{By} \\ \alpha_A \\ \alpha_B \end{pmatrix} = \text{Find}(T, a_{By}, \alpha_A, \alpha_B)$$

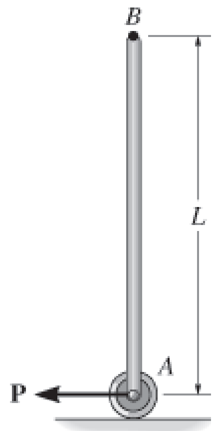
$T = 15.7\ \text{N}$

$$\begin{pmatrix} \alpha_A \\ \alpha_B \end{pmatrix} = \begin{pmatrix} 43.6 \\ 43.6 \end{pmatrix} \frac{\text{rad}}{\text{s}^2}$$

$a_{By} = -7.848\ \frac{\text{m}}{\text{s}^2}$

Problem 17-105

The uniform bar of mass m and length L is balanced in the vertical position when the horizontal force **P** is applied to the roller at A. Determine the bar's initial angular acceleration and the acceleration of its top point B.



Solution:

$$-P = m a_x \qquad a_x = \frac{-P}{m}$$

$$-P\left(\frac{L}{2}\right) = \frac{1}{12} m L^2 \alpha \qquad \alpha = \frac{-6P}{m L}$$

$$a_B = a_x - \alpha\left(\frac{L}{2}\right) \qquad a_B = \frac{2P}{m}$$

positive means to the right

Problem 17-106

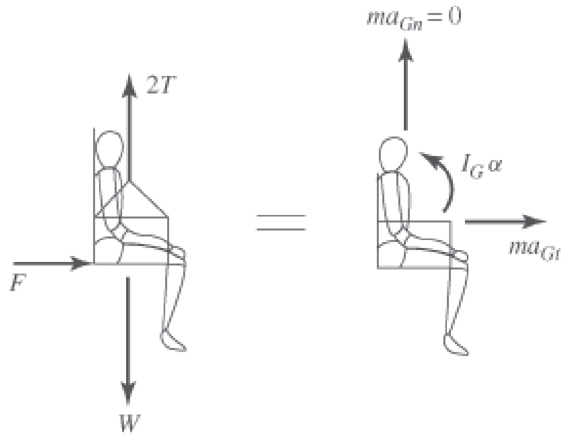
A woman sits in a rigid position in the middle of the swing. The combined weight of the woman and swing is W and the radius of gyration about the center of mass G is k_G. If a man pushes on the swing with a horizontal force **F** as shown, determine the initial angular acceleration and the tension in each of the two supporting chains AB. During the motion, assume that the chain segment CAD remains rigid. The swing is originally at rest.

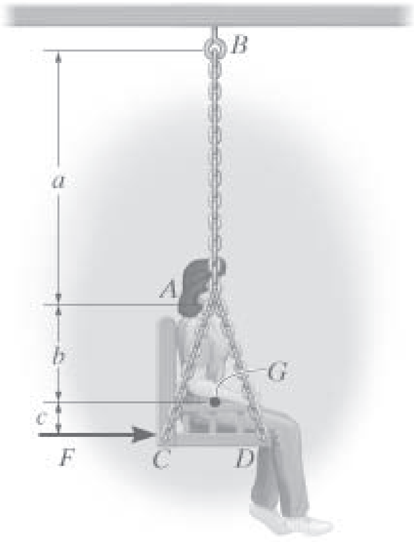
Given:

$W = 180\ \text{lb} \quad a = 4\ \text{ft} \quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$k_G = 2.5\ \text{ft} \quad b = 1.5\ \text{ft}$

$F = 20\ \text{lb} \quad c = 0.4\ \text{ft}$





Solution:

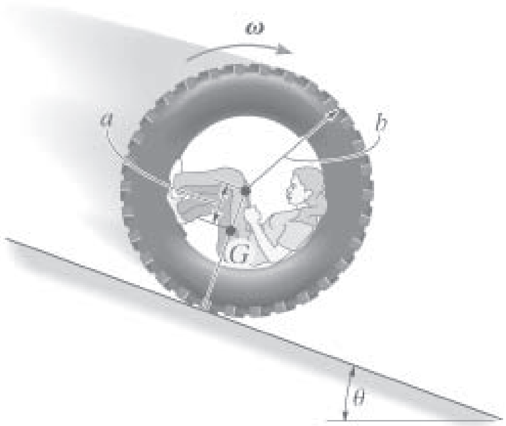
Guesses $\quad T = 1 \text{ lb} \qquad a_{Gt} = 1 \frac{\text{ft}}{\text{s}^2} \qquad \alpha = 1 \frac{\text{rad}}{\text{s}^2}$

Given $\quad F = \left(\frac{W}{g}\right) a_{Gt} \qquad 2T - W = 0 \qquad F c = \left(\frac{W}{g}\right) k_G^2 \alpha$

$$\begin{pmatrix} T \\ a_{Gt} \\ \alpha \end{pmatrix} = \text{Find}(T, a_{Gt}, \alpha) \qquad a_{Gt} = 3.58 \frac{\text{ft}}{\text{s}^2} \qquad \alpha = 0.229 \frac{\text{rad}}{\text{s}^2} \qquad T = 90.0 \text{ lb}$$

Problem 17-107

A girl sits snugly inside a large tire such that together the girl and tire have a total weight W, a center of mass at G, and a radius of gyration k_G about G. If the tire rolls freely down the incline, determine the normal and frictional forces it exerts on the ground when it is in the position shown and has an angular velocity ω. Assume that the tire does not slip as it rolls.



Given:

$W = 185 \text{ lb} \qquad b = 2 \text{ ft}$

$k_G = 1.65 \text{ ft} \qquad a = 0.75 \text{ ft}$

$\omega = 6 \frac{\text{rad}}{\text{s}} \qquad \theta = 20 \text{ deg}$

Solution:

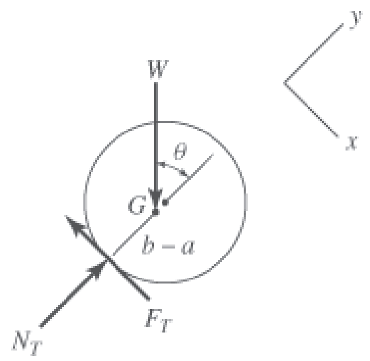
Guesses $\quad N_T = 1 \text{ lb} \qquad F_T = 1 \text{ lb}$

$\alpha = 1 \frac{\text{rad}}{\text{s}^2}$

Given

$$N_T - W\cos(\theta) = \frac{W}{g}a\omega^2 \qquad F_T(b-a) = \frac{W}{g}k_G{}^2\alpha$$

$$F_T - W\sin(\theta) = \frac{-W}{g}(b-a)\alpha$$



$$\begin{pmatrix} N_T \\ F_T \\ \alpha \end{pmatrix} = \mathrm{Find}(N_T, F_T, \alpha) \qquad \alpha = 3.2\,\frac{\mathrm{rad}}{\mathrm{s}^2}$$

$$\begin{pmatrix} N_T \\ F_T \end{pmatrix} = \begin{pmatrix} 329.0 \\ 40.2 \end{pmatrix}\mathrm{lb}$$

***Problem 17-108**

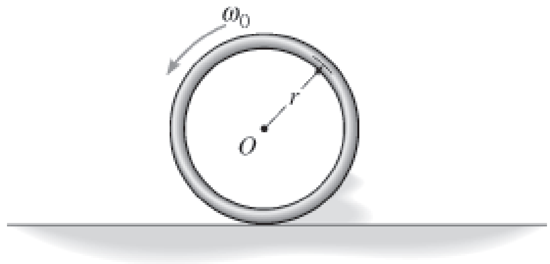
The hoop or thin ring of weight W is given an initial angular velocity ω_0 when it is placed on the surface. If the coefficient of kinetic friction between the hoop and the surface is μ_k, determine the distance the hoop moves before it stops slipping.

Given:

$$W = 10\ \mathrm{lb} \qquad \mu_k = 0.3$$

$$\omega_0 = 6\,\frac{\mathrm{rad}}{\mathrm{s}} \qquad r = 6\ \mathrm{in}$$

$$g = 32.2\,\frac{\mathrm{ft}}{\mathrm{s}^2}$$

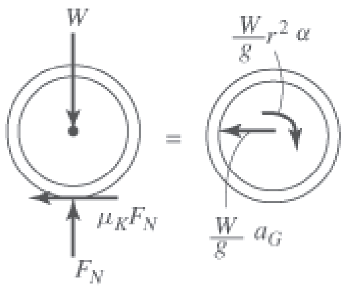


Solution:

$$F_N - W = 0 \qquad F_N = W \qquad F_N = 10\,\mathrm{lb}$$

$$\mu_k F_N = \left(\frac{W}{g}\right)a_G \qquad a_G = \mu_k g \qquad a_G = 9.66\,\frac{\mathrm{ft}}{\mathrm{s}^2}$$

$$\mu_k F_N r = \left(\frac{W}{g}\right)r^2\alpha \qquad \alpha = \frac{\mu_k g}{r} \qquad \alpha = 19.32\,\frac{\mathrm{rad}}{\mathrm{s}^2}$$



When it stops slipping

$$v_G = \omega r$$

$$a_G t = (\omega_0 - \alpha t)r \qquad t = \frac{\omega_0 r}{a_G + \alpha r} \qquad t = 0.155\ \mathrm{s}$$

$d = \frac{1}{2} a_G t^2$ $\quad\quad$ $d = 1.398\ \text{in}$

Problem 17-109

The circular plate of weight W is suspended from a pin at A. If the pin is connected to a track which is given acceleration a_A, determine the horizontal and vertical components of reaction at A and the acceleration of the plate's mass center G. The plate is originally at rest.

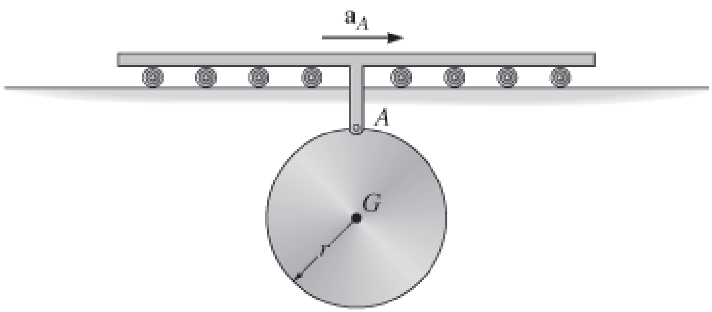
Given:

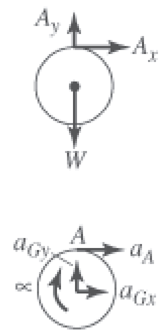
$W = 15\ \text{lb}$

$a_A = 3\ \frac{\text{ft}}{\text{s}^2}$

$r = 2\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$





Solution:

Guesses $\quad a_{Gx} = 1\ \frac{\text{ft}}{\text{s}^2} \quad \alpha = 1\ \frac{\text{rad}}{\text{s}^2} \quad A_x = 1\ \text{lb} \quad A_y = 1\ \text{lb}$

Given $\quad A_x = \frac{W}{g} a_{Gx} \quad A_y - W = 0 \quad -A_x r = \frac{-1}{2}\frac{W}{g} r^2 \alpha \quad a_{Gx} + \alpha r = a_A$

$$\begin{pmatrix} a_{Gx} \\ \alpha \\ A_x \\ A_y \end{pmatrix} = \text{Find}(a_{Gx}, \alpha, A_x, A_y) \quad \begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 0.466 \\ 15.000 \end{pmatrix} \text{lb} \quad a_{Gx} = 1\ \frac{\text{ft}}{\text{s}^2}$$

$$\alpha = 1\ \frac{\text{rad}}{\text{s}^2}$$

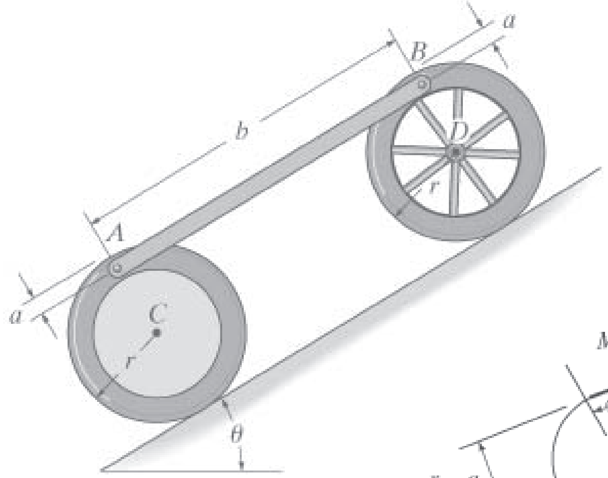
Problem 17-110

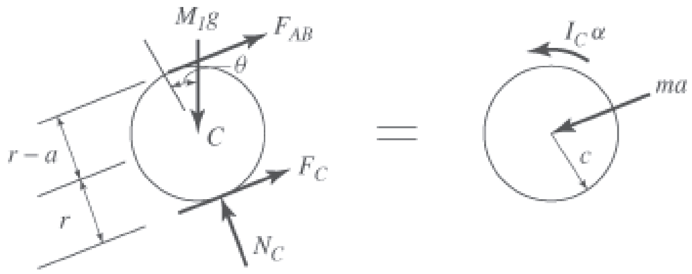
Wheel C has a mass M_1 and a radius of gyration k_C, whereas wheel D has a mass M_2 and a radius of gyration k_D. Determine the angular acceleration of each wheel at the instant shown. Neglect the mass of the link and assume that the assembly does not slip on the plane.

Given:

$M_1 = 60\ \text{kg} \quad k_C = 0.4\ \text{m} \quad r = 0.5\ \text{m} \quad b = 2\ \text{m}$

$M_2 = 40\ \text{kg} \quad k_D = 0.35\ \text{m} \quad a = 0.1\ \text{m} \quad \theta = 30\ \text{deg}$

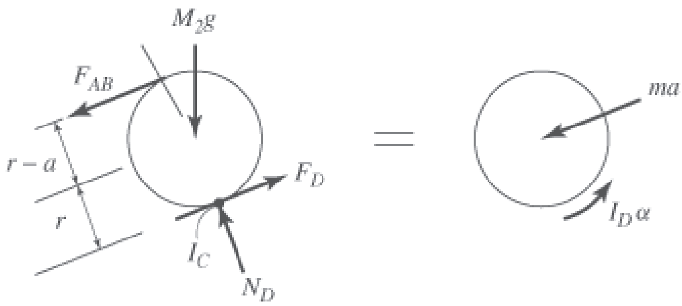




Solution:

Both wheels have the same angular acceleration.

Guesses



$$F_{AB} = 1\ \text{N}$$

$$\alpha = 1\ \frac{\text{rad}}{\text{s}^2}$$

Given

$$-F_{AB}(2r - a) + M_1 g \sin(\theta) r = M_1 k_C^2 \alpha + M_1(r\alpha)r$$

$$F_{AB}(2r - a) + M_2 g \sin(\theta) r = M_2 k_D^2 \alpha + M_2(r\alpha)r$$

$$\begin{pmatrix} F_{AB} \\ \alpha \end{pmatrix} = \text{Find}(F_{AB}, \alpha) \qquad F_{AB} = -6.21\ \text{N} \qquad \alpha = 6.21\ \frac{\text{rad}}{\text{s}^2}$$

Problem 17-111

The assembly consists of a disk of mass m_D and a bar of mass m_b which is pin connected to the disk. If the system is released from rest, determine the angular acceleration of the disk. The coefficients of static and kinetic friction between the disk and the inclined plane are μ_s and μ_k respectively. Neglect friction at B.

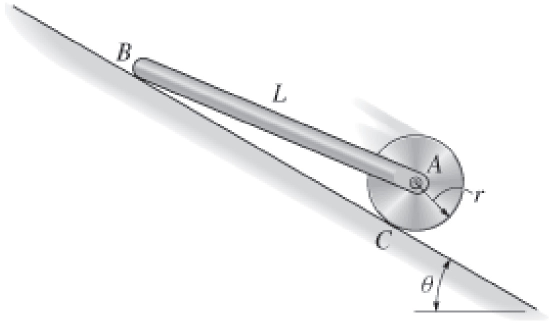
Given:

$m_D = 8\ \text{kg}$ $\quad L = 1\ \text{m}$

$m_b = 10\ \text{kg}$ $\quad r = 0.3\ \text{m}$

$\mu_s = 0.6$ $\quad \theta = 30\ \text{deg}$

$\mu_k = 0.4$ $\quad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$



Solution: $\quad \phi = \text{asin}\left(\dfrac{r}{L}\right)$

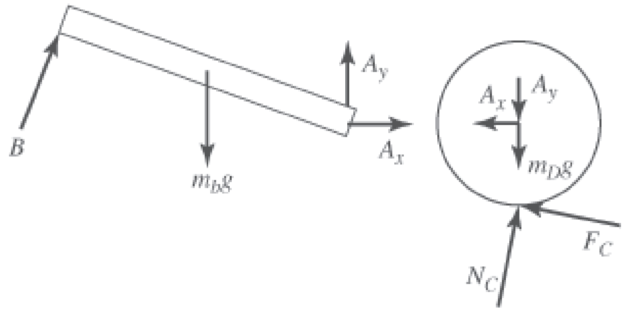
Assume no slip

Guesses

$N_C = 1\ \text{N} \quad F_C = 1\ \text{N}$

$\alpha = 1\ \dfrac{\text{rad}}{\text{s}^2} \quad a_A = 1\ \dfrac{\text{m}}{\text{s}^2}$

$F_{max} = 1\ \text{N}$



Given

$$N_C L\cos(\phi) - m_D g L\cos(\theta-\phi) - m_b g\frac{L}{2}\cos(\theta-\phi) = \frac{-1}{2}m_D r^2\alpha - m_D a_A r - m_b a_A\frac{r}{2}$$

$$-F_C + (m_D+m_b)g\sin(\theta) = (m_D+m_b)a_A$$

$$F_C r = \frac{1}{2}m_D r^2\alpha \qquad a_A = r\alpha \qquad F_{max} = \mu_s N_C$$

$$\begin{pmatrix} N_C \\ F_C \\ a_A \\ \alpha \\ F_{max} \end{pmatrix} = \text{Find}(N_C, F_C, a_A, \alpha, F_{max}) \qquad \begin{pmatrix} N_C \\ F_C \\ F_{max} \end{pmatrix} = \begin{pmatrix} 109.042 \\ 16.053 \\ 65.425 \end{pmatrix}\text{N} \qquad \alpha = 13.377\ \frac{\text{rad}}{\text{s}^2}$$

Since $F_C = 16.053\ \text{N} < F_{max} = 65.425\ \text{N}$ then our no-slip assumption is correct.

Problem 17-112

The assembly consists of a disk of mass m_D and a bar of mass m_b which is pin connected to the disk. If the system is released from rest, determine the angular acceleration of the disk. The coefficients of static and kinetic friction between the disk and the inclined plane are μ_s and μ_k respectively. Neglect friction at B. Solve if the bar is removed.

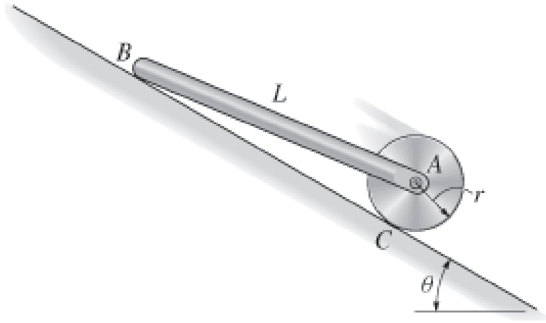
Given:

$m_D = 8\ \text{kg}$ $\quad L = 1\ \text{m}$

$m_b = 0\ \text{kg}$ $\quad r = 0.3\ \text{m}$

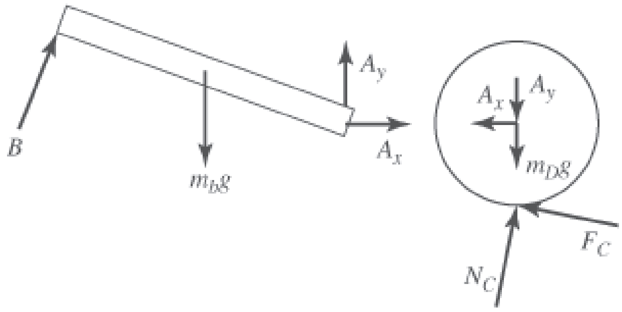
$\mu_s = 0.15$ $\quad \theta = 30\ \text{deg}$

$\mu_k = 0.1$ $\quad g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution: $\quad \phi = \operatorname{asin}\left(\frac{r}{L}\right)$

Assume no slip



Guesses

$N_C = 1\ \text{N}$ $\quad F_C = 1\ \text{N}$

$\alpha = 1\ \frac{\text{rad}}{\text{s}^2}$ $\quad a_A = 1\ \frac{\text{m}}{\text{s}^2}$

$F_{max} = 1\ \text{N}$

Given

$$N_C L\cos(\phi) - m_D g L\cos(\theta-\phi) - m_b g\frac{L}{2}\cos(\theta-\phi) = \frac{-1}{2}m_D r^2\alpha - m_D a_A r - m_b a_A\frac{r}{2}$$

$$-F_C + (m_D + m_b)g\sin(\theta) = (m_D + m_b)a_A$$

$$F_C r = \frac{1}{2}m_D r^2\alpha \qquad a_A = r\alpha \qquad F_{max} = \mu_s N_C$$

$$\begin{pmatrix} N_C \\ F_C \\ a_A \\ \alpha \\ F_{max} \end{pmatrix} = \operatorname{Find}(N_C, F_C, a_A, \alpha, F_{max}) \qquad \begin{pmatrix} N_C \\ F_C \\ F_{max} \end{pmatrix} = \begin{pmatrix} 67.966 \\ 13.08 \\ 10.195 \end{pmatrix}\text{N} \qquad \alpha = 10.9\ \frac{\text{rad}}{\text{s}^2}$$

Since $F_C = 13.08\ \text{N}$ $> F_{max} = 10.195\ \text{N}$ then our no-slip assumption is wrong and we know that slipping does occur.

Guesses

$N_C = 1\ \text{N}$ $\quad F_C = 1\ \text{N}$ $\quad \alpha = 1\ \frac{\text{rad}}{\text{s}^2}$ $\quad a_A = 1\ \frac{\text{m}}{\text{s}^2}$ $\quad F_{max} = 1\ \text{N}$

Given

$$N_C L\cos(\phi) - m_D g L\cos(\theta - \phi) - m_b g\frac{L}{2}\cos(\theta - \phi) = \frac{-1}{2}m_D r^2\alpha - m_D a_A r - m_b a_A \frac{r}{2}$$

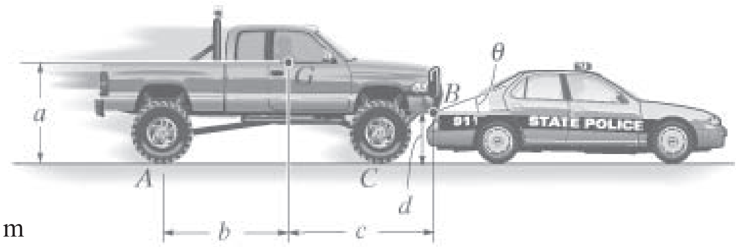
$$-F_C + (m_D + m_b)g\sin(\theta) = (m_D + m_b)a_A$$

$$F_C r = \frac{1}{2}m_D r^2\alpha \qquad F_{max} = \mu_s N_C \qquad F_C = \mu_k N_C$$

$$\begin{pmatrix} N_C \\ F_C \\ a_A \\ \alpha \\ F_{max} \end{pmatrix} = \text{Find}(N_C, F_C, a_A, \alpha, F_{max}) \qquad \begin{pmatrix} N_C \\ F_C \\ F_{max} \end{pmatrix} = \begin{pmatrix} 67.966 \\ 6.797 \\ 10.195 \end{pmatrix} \text{N} \qquad \alpha = 5.664\frac{\text{rad}}{\text{s}^2}$$

Problem 17-113

A "lifted" truck can become a road hazard since the bumper is high enough to ride up a standard car in the event the car is rear-ended. As a model of this case consider the truck to have a mass M, a mass center G, and a radius of gyration k_G about G. Determine the horizontal and vertical components of acceleration of the mass center G, and the angular acceleration of the truck, at the moment its front wheels at C have just left the ground and its smooth front bumper begins to ride up the back of the stopped car so that point B has a velocity of v_B at angle θ from the horizontal. Assume the wheels are free to roll, and neglect the size of the wheels and the deformation of the material.



Units Used:

$\text{Mg} = 10^3\ \text{kg}$

$\text{kN} = 10^3\ \text{N}$

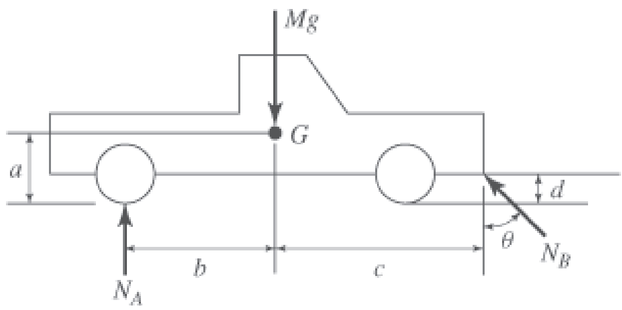
Given:

$M = 2.70\ \text{Mg} \quad a = 1.3\ \text{m}$

$\theta = 20\ \text{deg} \quad b = 1.6\ \text{m}$

$k_G = 1.45\ \text{m} \quad c = 1.2\ \text{m}$

$v_B = 8\ \frac{\text{m}}{\text{s}} \quad d = 0.4\ \text{m}$

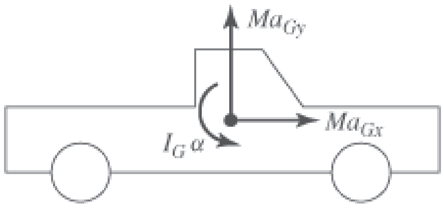


Solution:

Guesses $v_A = 1\ \frac{\text{m}}{\text{s}} \quad \omega = 1\ \frac{\text{rad}}{\text{s}}$

$a_B = 1\ \frac{\text{m}}{\text{s}^2} \quad \alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

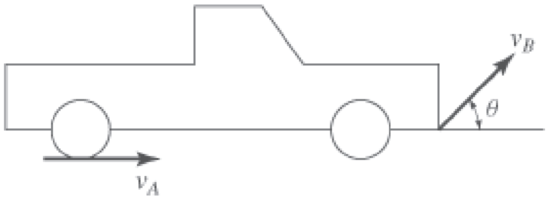
$a_{Gx} = 1\ \frac{\text{m}}{\text{s}^2}$ $\quad a_{Gy} = 1\ \frac{\text{m}}{\text{s}^2}$



$N_A = 1\ \text{N}$ $\quad N_B = 1\ \text{N}$

$a_A = 1\ \frac{\text{m}}{\text{s}^2}$

Given



$$N_A + N_B\cos(\theta) - Mg = Ma_{Gy}$$

$$-N_B\sin(\theta) = Ma_{Gx}$$

$$N_B\cos(\theta)c - N_B\sin(\theta)(a-d) - N_A b = Mk_G^2\alpha$$

$$\begin{pmatrix} v_A \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} b+c \\ d \\ 0 \end{pmatrix} = \begin{pmatrix} v_B\cos(\theta) \\ v_B\sin(\theta) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} a_A \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} b+c \\ d \\ 0 \end{pmatrix} - \omega^2\begin{pmatrix} b+c \\ d \\ 0 \end{pmatrix} = \begin{pmatrix} a_B\cos(\theta) \\ a_B\sin(\theta) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} a_{Gx} \\ a_{Gy} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} c \\ -a+d \\ 0 \end{pmatrix} - \omega^2\begin{pmatrix} c \\ -a+d \\ 0 \end{pmatrix} = \begin{pmatrix} a_B\cos(\theta) \\ a_B\sin(\theta) \\ 0 \end{pmatrix}$$

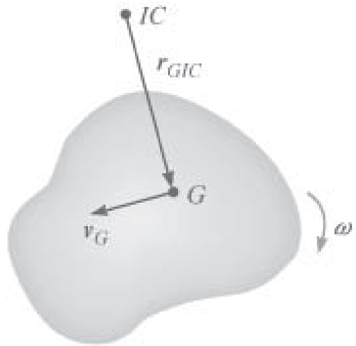
$$\begin{pmatrix} v_A \\ \omega \\ a_A \\ a_B \\ a_{Gx} \\ a_{Gy} \\ \alpha \\ N_A \\ N_B \end{pmatrix} = \text{Find}(v_A, \omega, a_A, a_B, a_{Gx}, a_{Gy}, \alpha, N_A, N_B)$$

$$\begin{pmatrix} a_{Gx} \\ a_{Gy} \end{pmatrix} = \begin{pmatrix} -1.82 \\ -1.69 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

$$\omega = 0.977\ \frac{\text{rad}}{\text{s}} \qquad \alpha = -0.283\ \frac{\text{rad}}{\text{s}^2}$$

$$\begin{pmatrix} a_A \\ a_B \end{pmatrix} = \begin{pmatrix} -0.664 \\ -3.431 \end{pmatrix} \frac{\text{m}}{\text{s}^2} \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 8.38 \\ 14.40 \end{pmatrix} \text{kN}$$

Problem 18-1



At a given instant the body of mass m has an angular velocity ω and its mass center has a velocity $\mathbf{v}_G$. Show that its kinetic energy can be represented as $T = 1/2\, I_{IC}\omega^2$, where I_{IC} is the moment of inertia of the body computed about the instantaneous axis of zero velocity, located a distance r_{GIC} from the mass center as shown.

Solution:

$$T = \left(\frac{1}{2}\right) m\, v_G^{\,2} + \left(\frac{1}{2}\right)(I_G)\,\omega^2$$ where $v_G = \omega\, r_{GIC}$

$$T = \left(\frac{1}{2}\right) m\left(\omega r_{GIC}\right)^2 + \frac{1}{2} I_G \omega^2$$

$$T = \left(\frac{1}{2}\right)\left(m\, r_{GIC}^{\,2} + I_G\right)\omega^2$$ However $m(r_{GIC})^2 + I_G = I_{IC}$

$$\boxed{T = \left(\frac{1}{2}\right) I_{IC}\omega^2}$$

Problem 18-2

The wheel is made from a thin ring of mass m_{ring} and two slender rods each of mass m_{rod}. If the torsional spring attached to the wheel's center has stiffness k, so that the torque on the center of the wheel is $M = -k\theta$, determine the maximum angular velocity of the wheel if it is rotated two revolutions and then released from rest.

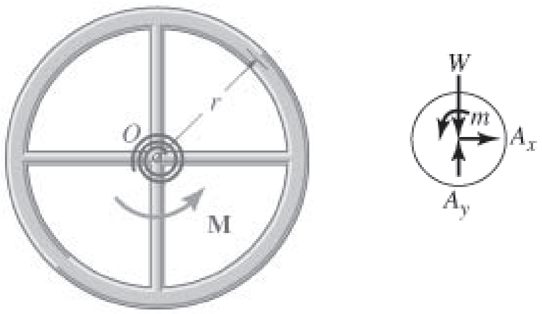
Given:

$m_{ring} = 5\ \text{kg}$

$m_{rod} = 2\ \text{kg}$

$k = 2\ \dfrac{\text{N}\cdot\text{m}}{\text{rad}}$

$r = 0.5\ \text{m}$



Solution:

$$I_O = 2\left[\frac{1}{12} m_{rod}(2r)^2\right] + m_{ring}\, r^2$$

$$I_O = 1.583\ \text{kg}\cdot\text{m}^2$$

$$T_1 + \Sigma U_{12} = T_2$$

$$0+\int_{4\pi}^{0} -k\,\theta\,d\theta = \frac{1}{2} I_O \omega^2 \qquad \omega = \sqrt{\frac{k}{I_O}}\,4\pi \qquad \omega = 14.1\,\frac{\text{rad}}{\text{s}}$$

Problem 18-3

At the instant shown, the disk of weight W has counterclockwise angular velocity ω when its center has velocity v. Determine the kinetic energy of the disk at this instant.

Given:

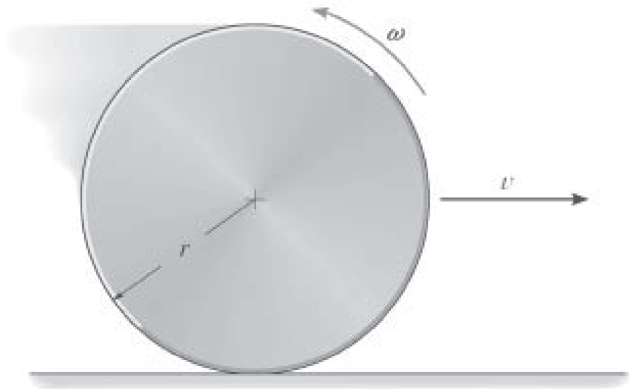
$W = 30$ lb

$\omega = 5\,\frac{\text{rad}}{\text{s}}$

$v = 20\,\frac{\text{ft}}{\text{s}}$

$r = 2$ ft

$g = 32.2\,\frac{\text{ft}}{\text{s}^2}$

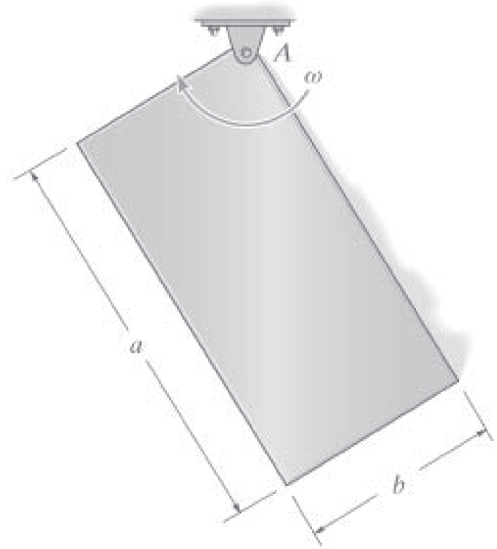


Solution:

$$T = \frac{1}{2}\left(\frac{1}{2}\frac{W}{g}r^2\right)\omega^2 + \frac{1}{2}\left(\frac{W}{g}\right)v^2 \qquad T = 210\,\text{ft}\cdot\text{lb}$$

***Problem 18-4**

The uniform rectangular plate has weight W. If the plate is pinned at A and has an angular velocity ω, determine the kinetic energy of the plate.



Given:

$W = 30$ lb

$\omega = 3\,\frac{\text{rad}}{\text{s}}$

$a = 2$ ft

$b = 1$ ft

Solution:

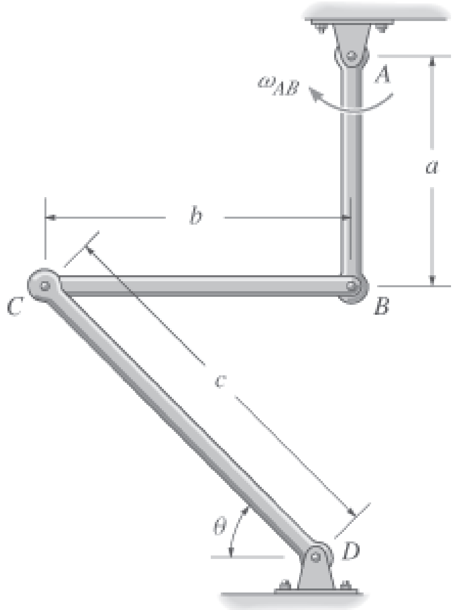
$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$$

$$T = \frac{1}{2}\left(\frac{W}{g}\right)\left(\omega\frac{\sqrt{b^2+a^2}}{2}\right)^2 + \frac{1}{2}\left[\frac{1}{12}\left(\frac{W}{g}\right)\left(b^2+a^2\right)\right]\omega^2$$

$T = 6.99\ \text{ft}\cdot\text{lb}$

Problem 18-5

At the instant shown, link AB has angular velocity ω_{AB}. If each link is considered as a uniform slender bar with weight density γ, determine the total kinetic energy of the system.



Given:

$\omega_{AB} = 2\ \frac{\text{rad}}{\text{s}}$ $\quad a = 3\ \text{in}$

$\gamma = 0.5\ \frac{\text{lb}}{\text{in}}$ $\quad b = 4\ \text{in}$

$\theta = 45\ \text{deg}$ $\quad c = 5\ \text{in}$

Solution: $\rho = \frac{\gamma}{g}$

Guesses

$\omega_{BC} = 1\ \frac{\text{rad}}{\text{s}}$ $\quad \omega_{CD} = 1\ \frac{\text{rad}}{\text{s}}$

$v_{Gx} = 1\ \frac{\text{in}}{\text{s}}$ $\quad v_{Gy} = 1\ \frac{\text{in}}{\text{s}}$ $\quad T = 1\ \text{lb}\cdot\text{ft}$

Given

$$\begin{pmatrix}0\\0\\-\omega_{AB}\end{pmatrix}\times\begin{pmatrix}0\\-a\\0\end{pmatrix}+\begin{pmatrix}0\\0\\\omega_{BC}\end{pmatrix}\times\begin{pmatrix}-b\\0\\0\end{pmatrix}+\begin{pmatrix}0\\0\\\omega_{CD}\end{pmatrix}\times\begin{pmatrix}c\cos(\theta)\\-c\sin(\theta)\\0\end{pmatrix} = 0$$

$$\begin{pmatrix}0\\0\\-\omega_{AB}\end{pmatrix}\times\begin{pmatrix}0\\-a\\0\end{pmatrix}+\begin{pmatrix}0\\0\\\omega_{BC}\end{pmatrix}\times\begin{pmatrix}\frac{-b}{2}\\0\\0\end{pmatrix} = \begin{pmatrix}v_{Gx}\\v_{Gy}\\0\end{pmatrix}$$

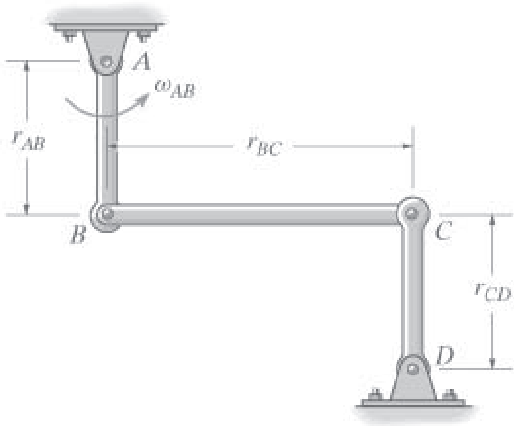
$$T = \frac{1}{2}\left(\frac{\rho a^3}{3}\right)\omega_{AB}^2 + \frac{1}{2}\left(\frac{\rho b^3}{12}\right)\omega_{BC}^2 + \frac{1}{2}\rho b\left(v_{Gx}^2+v_{Gy}^2\right) + \frac{1}{2}\left(\frac{\rho c^3}{3}\right)\omega_{CD}^2$$

$$\begin{pmatrix} \omega_{BC} \\ \omega_{CD} \\ v_{Gx} \\ v_{Gy} \\ T \end{pmatrix} = \text{Find}(\omega_{BC}, \omega_{CD}, v_{Gx}, v_{Gy}, T) \qquad \begin{pmatrix} \omega_{BC} \\ \omega_{CD} \end{pmatrix} = \begin{pmatrix} 1.5 \\ 1.697 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad \begin{pmatrix} v_{Gx} \\ v_{Gy} \end{pmatrix} = \begin{pmatrix} -0.5 \\ -0.25 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

$T = 0.0188\ \text{ft}\cdot\text{lb}$

Problem 18-6

Determine the kinetic energy of the system of three links. Links AB and CD each have weight W_1, and link BC has weight W_2.



Given:

$W_1 = 10\ \text{lb}$

$W_2 = 20\ \text{lb}$

$\omega_{AB} = 5\ \frac{\text{rad}}{\text{s}}$

$r_{AB} = 1\ \text{ft}$

$r_{BC} = 2\ \text{ft}$

$r_{CD} = 1\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$\omega_{BC} = 0\ \frac{\text{rad}}{\text{s}} \qquad \omega_{CD} = \omega_{AB}\left(\frac{r_{AB}}{r_{CD}}\right)$

$$T = \frac{1}{2}\left(\frac{W_1}{g}\right)\left(\frac{r_{AB}^2}{3}\right)\omega_{AB}^2 + \frac{1}{2}\left(\frac{W_2}{g}\right)(\omega_{AB} r_{AB})^2 + \frac{1}{2}\left(\frac{W_1}{g}\right)\frac{r_{CD}^2}{3}\omega_{CD}^2$$

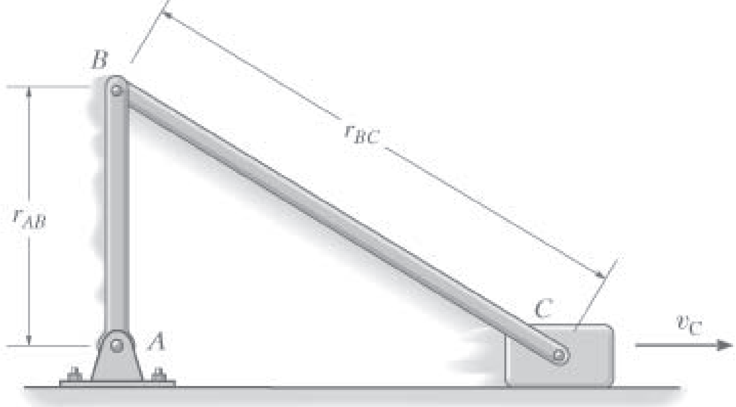
$T = 10.4\ \text{ft}\cdot\text{lb}$

Problem 18-7

The mechanism consists of two rods, AB and BC, which have weights W_1 and W_2, respectively, and a block at C of weight W_3. Determine the kinetic energy of the system at the instant shown, when the block is moving at speed v_C.

Given:

$W_1 = 10\ \text{lb}$



$W_2 = 20\ \text{lb}$

$W_3 = 4\ \text{lb}$

$r_{AB} = 2\ \text{ft}$

$r_{BC} = 4\ \text{ft}$

$v_C = 3\ \frac{\text{ft}}{\text{s}}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$\omega_{BC} = 0\ \frac{\text{rad}}{\text{s}}$ $\qquad \omega_{AB} = \frac{v_C}{r_{AB}}$

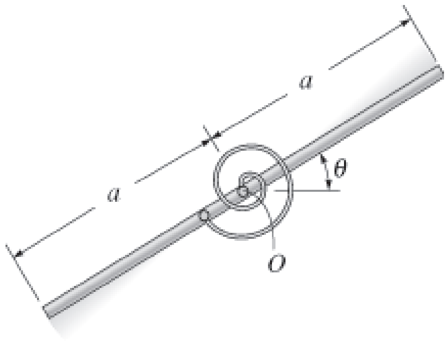
$$T = \frac{1}{2}\left(\frac{W_1}{g}\right)\left(\frac{r_{AB}^2}{3}\right)\omega_{AB}^2 + \frac{1}{2}\left(\frac{W_2}{g}\right)v_C^2 + \frac{1}{2}\left(\frac{W_3}{g}\right)v_C^2 \qquad T = 3.82\ \text{lb}\cdot\text{ft}$$

***Problem 18-8**

The bar of weight W is pinned at its center O and connected to a torsional spring. The spring has a stiffness k, so that the torque developed is $M = k\theta$. If the bar is released from rest when it is vertical at $\theta = 90°$, determine its angular velocity at the instant $\theta = 0°$. Use the prinicple of work and energy.

Given:

$W = 10\ \text{lb}$



$k = 5\ \frac{\text{lb}\cdot\text{ft}}{\text{rad}}$

$a = 1\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$\theta_0 = 90\ \text{deg} \qquad \theta_f = 0\ \text{deg}$

Guess $\omega = 1\ \frac{\text{rad}}{\text{s}}$

Given $\frac{1}{2}k\theta_0^2 = \frac{1}{2}k\theta_f^2 + \frac{1}{2}\left(\frac{W}{g}\right)\frac{(2a)^2}{12}\omega^2$

$\omega = \text{Find}(\omega)$ $\omega = 10.9\ \frac{\text{rad}}{\text{s}}$

Problem 18-9

A force P is applied to the cable which causes the reel of mass M to turn since it is resting on the two rollers A and B of the dispenser. Determine the angular velocity of the reel after it has made two revolutions starting from rest. Neglect the mass of the rollers and the mass of the cable. The radius of gyration of the reel about its center axis is k_G.

Given:

$P = 20\ \text{N}$

$M = 175\ \text{kg}$

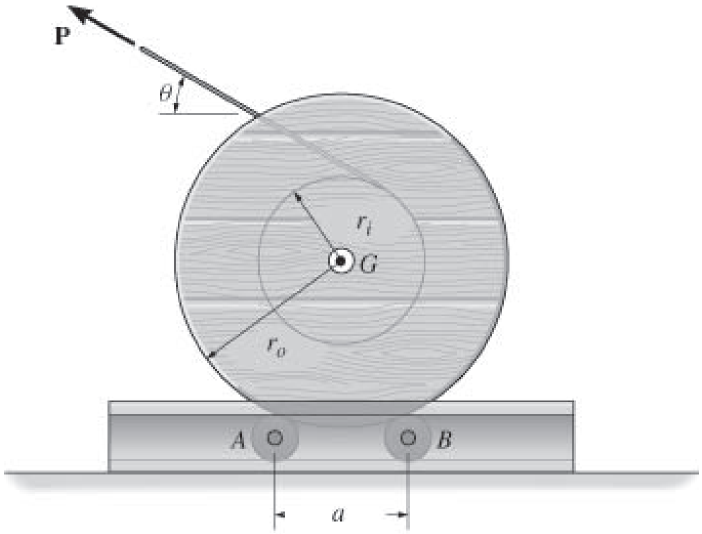
$k_G = 0.42\ \text{m}$

$\theta = 30\ \text{deg}$

$r_i = 250\ \text{mm}$

$r_0 = 500\ \text{mm}$

$a = 400\ \text{mm}$



Solution:

$$0 + P2(2\pi r_i) = \frac{1}{2}Mk_G^2\,\omega^2$$

$$\omega = \sqrt{\frac{8\pi P r_i}{M k_G^2}} \qquad \omega = 2.02\ \frac{\text{rad}}{\text{s}}$$

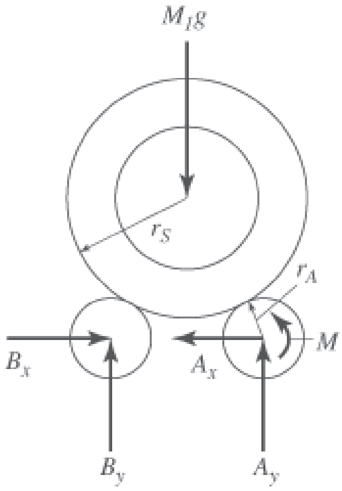
Problem 18-10

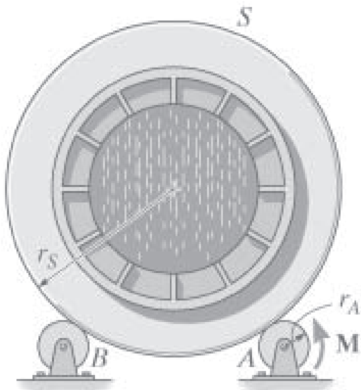
The rotary screen S is used to wash limestone. When empty it has a mass M_1 and a radius of gyration k_G. Rotation is achieved by applying a torque M about the drive wheel A. If no slipping occurs at A and the supporting wheel at B is free to roll, determine the angular velocity of the screen after it has rotated n revolutions. Neglect the mass of A and B.

Unit Used:

$\text{rev} = 2\pi \text{ rad}$

Given:





$M_1 = 800 \text{ kg}$

$k_G = 1.75 \text{ m}$

$M = 280 \text{ N·m}$

$r_S = 2 \text{ m}$

$r_A = 0.3 \text{ m}$

$n = 5 \text{ rev}$

Solution:

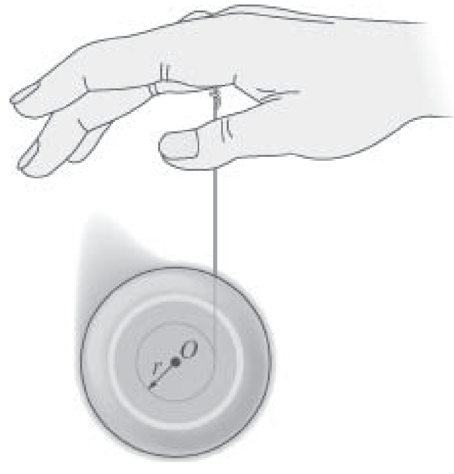
$$M\left(\frac{r_S}{r_A}n\right) = \frac{1}{2}M_1 k_G^2 \omega^2$$

$$\omega = \sqrt{\frac{2M r_S n}{r_A M_1 k_G^2}} \qquad \omega = 6.92 \frac{\text{rad}}{\text{s}}$$

Problem 18-11

A yo-yo has weight W and radius of gyration k_O. If it is released from rest, determine how far it must descend in order to attain angular velocity ω. Neglect the mass of the string and assume that the string is wound around the central peg such that the mean radius at which it unravels is r.

Given:



$W = 0.3 \text{ lb}$

$k_O = 0.06 \text{ ft}$

$\omega = 70 \frac{\text{rad}}{\text{s}}$

$r = 0.02 \text{ ft}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$

Solution:

$$0 + W h = \frac{1}{2}\left(\frac{W}{g}\right)(r\omega)^2 + \frac{1}{2}\left(\frac{W}{g}k_O^2\right)\omega^2$$

$$h = \frac{r^2 + k_O^2}{2g}\omega^2 \qquad h = 0.304\ \text{ft}$$

***Problem 18-12**

The soap-box car has weight W_c including the passenger but *excluding* its four wheels. Each wheel has weight W_w, radius r, and radius of gyration k, computed about an axis passing through the wheel's axle. Determine the car's speed after it has traveled a distance d starting from rest. The wheels roll without slipping. Neglect air resistance.

Given:

$W_c = 110\ \text{lb}$

$W_w = 5\ \text{lb}$

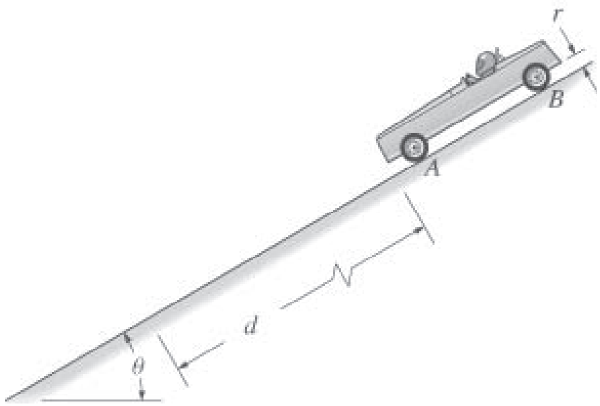
$r = 0.5\ \text{ft}$

$k = 0.3\ \text{ft}$

$d = 100\ \text{ft}$

$\theta = 30\ \text{deg}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution:

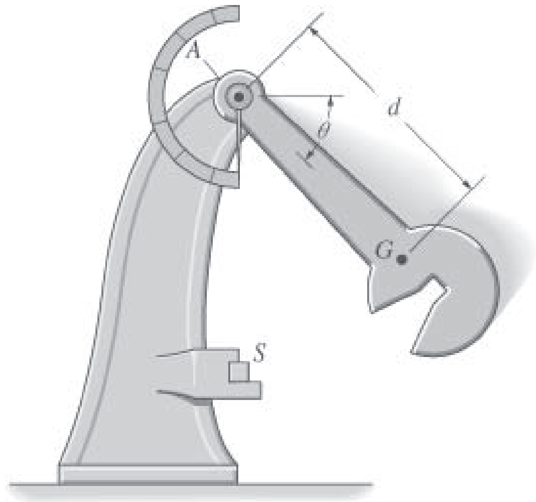
$$(W_c + 4W_w)d\sin(\theta) = \frac{1}{2}\left(\frac{W_c + 4W_w}{g}\right)v^2 + \frac{1}{2}4\left(\frac{W_w}{g}k^2\right)\left(\frac{v}{r}\right)^2$$

$$v = \sqrt{\frac{2(W_c + 4W_w)d\sin(\theta)g}{W_c + 4W_w + 4W_w\dfrac{k^2}{r^2}}} \qquad v = 55.2\ \frac{\text{ft}}{\text{s}}$$

Problem 18-13

The pendulum of the Charpy impact machine has mass M and radius of gyration k_A. If it is released from rest when $\theta = 0°$, determine its angular velocity just before it strikes the specimen S, $\theta = 90°$.

Given:



$M = 50\ \text{kg}$

$k_A = 1.75\ \text{m}$

$d = 1.25\ \text{m}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Solution:

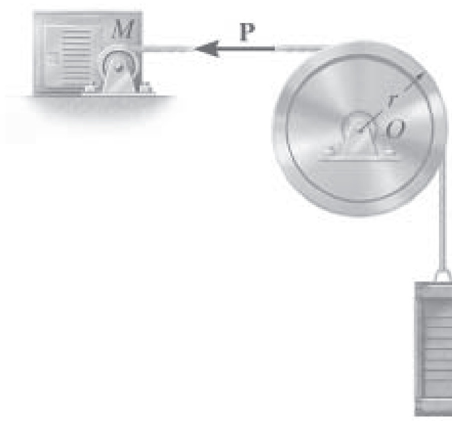
$$0 + M g d = \frac{1}{2} M k_A^2\, \omega_2^2$$

$$\omega_2 = \sqrt{\frac{2 g d}{k_A^2}} \qquad \omega_2 = 2.83\ \frac{\text{rad}}{\text{s}}$$

Problem 18-14

The pulley of mass M_p has a radius of gyration about O of k_O. If a motor M supplies a force to the cable of $P = a\,(b - ce^{-dx})$, where x is the amount of cable wound up, determine the speed of the crate of mass M_c when it has been hoisted a distance h starting from rest. Neglect the mass of the cable and assume the cable does not slip on the pulley.

Given:



$M_p = 10\ \text{kg}$	$a = 800\ \text{N}$
$M_c = 50\ \text{kg}$	$b = 3$
$k_O = 0.21\ \text{m}$	$c = 2$
$r = 0.3\ \text{m}$	$d = \dfrac{1}{\text{m}}$
$h = 2\ \text{m}$	

Solution:

Guesses $\quad v_c = 1\ \dfrac{\text{m}}{\text{s}}$

Given $\displaystyle\int_0^h a\left(b - c e^{-d x}\right) \mathrm{d}x = \frac{1}{2} M_p k_O^2 \left(\frac{v_C}{r}\right)^2 + \frac{1}{2} M_c v_C^2 + M_c g h$

$v_C = \mathrm{Find}(v_C)$ $\quad$ $v_C = 9.419 \ \frac{\mathrm{m}}{\mathrm{s}}$

Problem 18-15

The uniform pipe has a mass M and radius of gyration about the z axis of k_G. If the worker pushes on it with a horizontal force F, applied perpendicular to the pipe, determine the pipe's angular velocity when it has rotated through angle θ about the z axis, starting from rest. Assume the pipe does not swing.

Units Used:

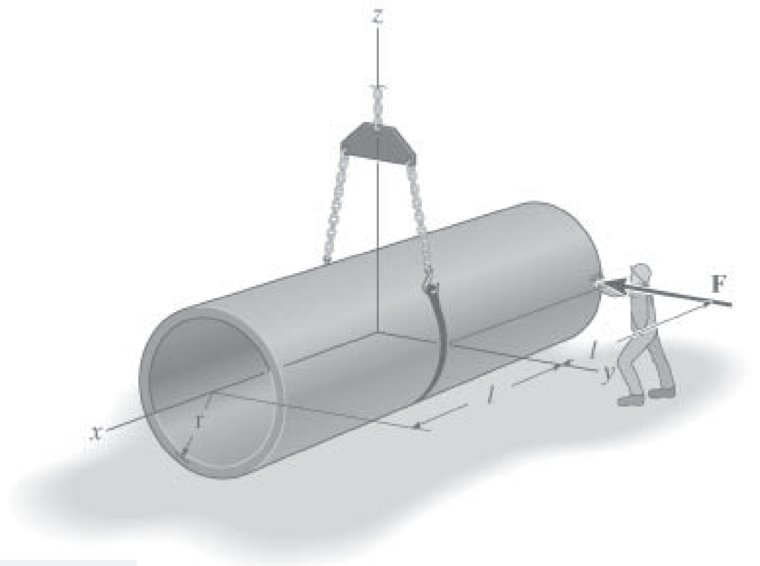
$\mathrm{Mg} = 10^3 \ \mathrm{kg}$

Given:

$M = 16 \ \mathrm{Mg}$ $\quad$ $\theta = 90 \ \mathrm{deg}$

$k_G = 2.7 \ \mathrm{m}$ $\quad$ $r = 0.75 \ \mathrm{m}$

$F = 50 \ \mathrm{N}$ $\quad$ $l = 3 \ \mathrm{m}$



Solution:

$$0 + F l \theta = \frac{1}{2} M k_G^2 \omega^2$$

$\omega = \dfrac{1}{M} \dfrac{\sqrt{M F l \pi}}{k_G}$ $\quad$ $\omega = 0.0636 \ \frac{\mathrm{rad}}{\mathrm{s}}$

***Problem 18-16**

The slender rod of mass m_{rod} is subjected to the force and couple moment. When it is in the position shown it has angular velocity ω_1. Determine its angular velocity at the instant it has rotated downward 90°. The force is always applied perpendicular to the axis of the rod. Motion occurs in the vertical plane.

Given:

$m_{rod} = 4\ \text{kg}$

$\omega_1 = 6\ \frac{\text{rad}}{\text{s}}$

$F = 15\ \text{N}$

$M = 40\ \text{N}\cdot\text{m}$

$a = 3\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution: Guess $\omega_2 = 1\ \frac{\text{rad}}{\text{s}}$

Given

$$\frac{1}{2}\left(\frac{m_{rod}a^2}{3}\right)\omega_1^{\;2} + F\,a\left(\frac{\pi}{2}\right) + m_{rod}g\left(\frac{a}{2}\right) + M\frac{\pi}{2} = \frac{1}{2}\left(\frac{m_{rod}a^2}{3}\right)\omega_2^{\;2}$$

$\omega_2 = \text{Find}(\omega_2)$ $\omega_2 = 8.25\ \frac{\text{rad}}{\text{s}}$

Problem 18-17

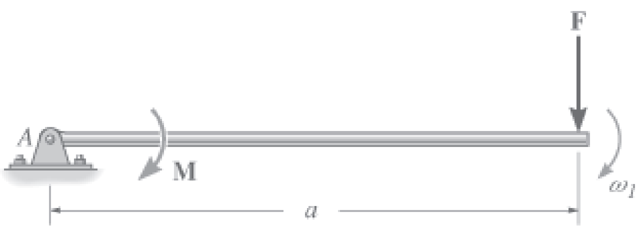
The slender rod of mass M is subjected to the force and couple moment. When the rod is in the position shown it has angular velocity ω_1. Determine its angular velocity at the instant it has rotated 360°. The force is always applied perpendicular to the axis of the rod and motion occurs in the vertical plane.

Given:

$m_{rod} = 4\ \text{kg}$ $M = 40\ \text{N}\cdot\text{m}$

$\omega_1 = 6\ \frac{\text{rad}}{\text{s}}$ $a = 3\ \text{m}$

$F = 15\ \text{N}$ $g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution:

Guess $\omega_2 = 1\ \frac{\text{rad}}{\text{s}}$

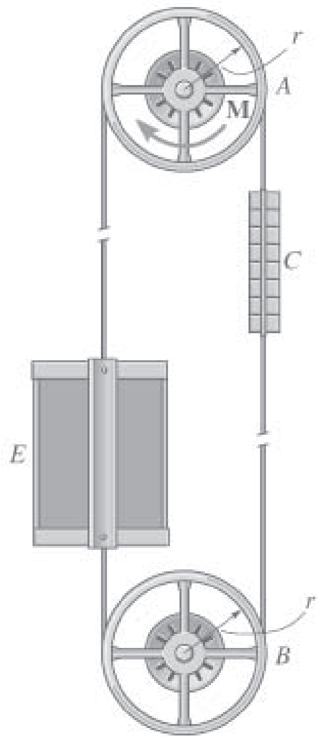
Given

$$\frac{1}{2}\left(\frac{m_{rod}a^2}{3}\right)\omega_1^{\,2} + F\,a2\pi + M2\pi = \frac{1}{2}\left(\frac{m_{rod}a^2}{3}\right)\omega_2^{\,2}$$

$\omega_2 = \text{Find}(\omega_2)$ $\qquad$ $\omega_2 = 11.2\ \frac{\text{rad}}{\text{s}}$

Problem 18-18

The elevator car E has mass m_E and the counterweight C has mass m_C. If a motor turns the driving sheave A with constant torque M, determine the speed of the elevator when it has ascended a distance d starting from rest. Each sheave A and B has mass m_S and radius of gyration k about its mass center or pinned axis. Neglect the mass of the cable and assume the cable does not slip on the sheaves.



Units Used: $\quad \text{Mg} = 1000\ \text{kg}$

Given:

$m_E = 1.80\ \text{Mg}$ $\qquad$ $d = 10\ \text{m}$

$m_C = 2.30\ \text{Mg}$ $\qquad$ $r = 0.35\ \text{m}$

$m_S = 150\ \text{kg}$ $\qquad$ $k = 0.2\ \text{m}$

$M = 100\ \text{N}\cdot\text{m}$ $\qquad$ $g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:

$$M\frac{d}{r} - (m_E - m_C)g\,d = \frac{1}{2}\left(m_E + m_C + 2m_S\frac{k^2}{r^2}\right)v^2$$

$$v = \sqrt{\frac{2\left[M\frac{d}{r} - (m_E - m_C)g\,d\right]}{m_E + m_C + 2m_S\frac{k^2}{r^2}}} \qquad v = 4.973\ \frac{\text{m}}{\text{s}}$$

Problem 18-19

The elevator car E has mass m_E and the counterweight C has mass m_C. If a motor turns the driving sheave A with torque $a\theta^2 + b$, determine the speed of the elevator when it has ascended a distance d starting from rest. Each sheave A and B has mass m_S and radius of gyration k about its mass center or pinned axis. Neglect the mass of the cable and assume the cable does not slip on the sheaves.

Units Used:

$Mg = 1000\ kg$

Given:

$m_E = 1.80\ Mg$

$m_C = 2.30\ Mg$

$m_S = 150\ kg$

$a = 0.06\ N\cdot m$

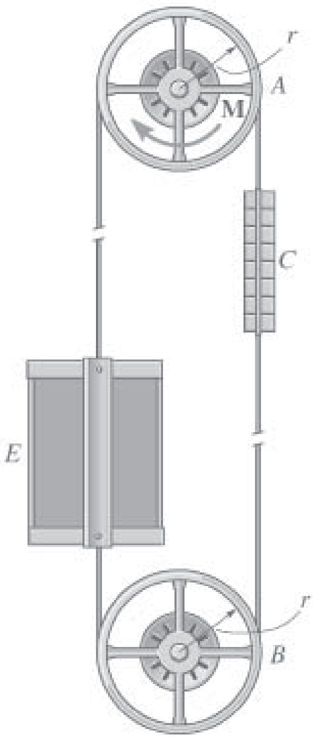
$b = 7.5\ N\cdot m$

$d = 12\ m$

$r = 0.35\ m$

$k = 0.2\ m$

$g = 9.81\ \frac{m}{s^2}$



Solution:

Guess $v = 1\ \frac{m}{s}$

Given $\displaystyle\int_0^{\frac{d}{r}} a\theta^2 + b\, d\theta - (m_E - m_C)g\,d = \frac{1}{2}\left(m_E + m_C + 2m_S\frac{k^2}{r^2}\right)v^2$

$v = \text{Find}(v)$ $\quad v = 5.343\ \frac{m}{s}$

***Problem 18-20**

The wheel has a mass M_1 and a radius of gyration k_O. A motor supplies a torque $\mathbf{M} = (a\theta + b)$, about the drive shaft at O. Determine the speed of the loading car, which has a mass M_2, after it travels a distance $s = d$. Initially the car is at rest when $s = 0$ and $\theta = 0°$. Neglect the mass of the attached cable and the mass of the car's wheels.

Given:

$M_1 = 100 \text{ kg}$

$M_2 = 300 \text{ kg}$

$d = 4 \text{ m}$

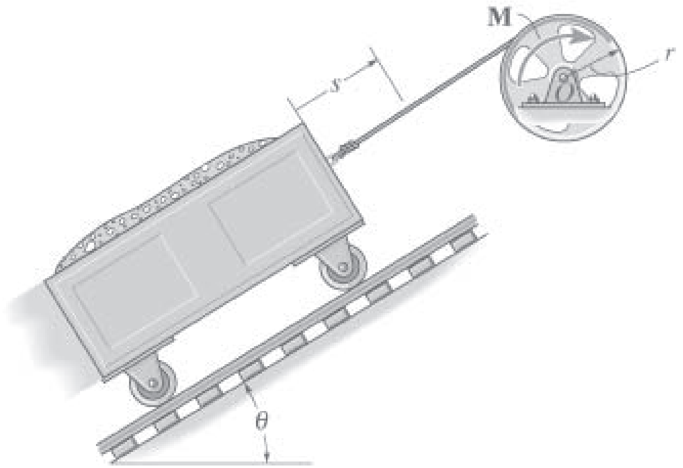
$a = 40 \text{ N·m}$

$b = 900 \text{ N·m}$

$r = 0.3 \text{ m}$

$k_O = 0.2 \text{ m}$

$\theta = 30 \text{ deg}$



Solution:

Guess $v = 1 \frac{\text{m}}{\text{s}}$

Given $$\int_0^{\frac{d}{r}} (a\theta + b)\, d\theta = \frac{1}{2}M_2 v^2 + \frac{1}{2}M_1 k_O^2\left(\frac{v}{r}\right)^2 + M_2 g\, d \sin(\theta)$$

$v = \text{Find}(v)$ $\quad v = 7.49 \frac{\text{m}}{\text{s}}$

Problem 18-21

The gear has a weight W and a radius of gyration k_G. If the spring is unstretched when the torque M is applied, determine the gear's angular velocity after its mass center G has moved to the left a distance d.

Given:

$W = 15 \text{ lb}$

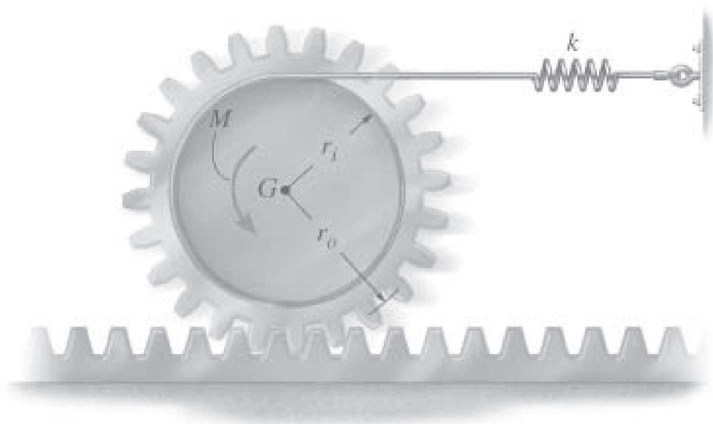
$M = 6 \text{ lb·ft}$

$r_o = 0.5 \text{ ft}$

$r_i = 0.4 \text{ ft}$

$d = 2 \text{ ft}$

$k = 3 \frac{\text{lb}}{\text{ft}}$



$k_G = 0.375\ \text{ft}$

Solution:

Guess $\omega = 1\ \dfrac{\text{rad}}{\text{s}}$

Given $$M\left(\frac{d}{r_o}\right) = \frac{1}{2}\left(\frac{W}{g}\right)\left(\omega r_o\right)^2 + \frac{1}{2}\left(\frac{W}{g}\right)k_G{}^2\,\omega^2 + \frac{1}{2}k\left(\frac{r_i + r_o}{r_o}d\right)^2$$

$\omega = \text{Find}(\omega)$ $\omega = 7.08\ \dfrac{\text{rad}}{\text{s}}$

Problem 18-22

The disk of mass m_d is originally at rest, and the spring holds it in equilibrium. A couple moment M is then applied to the disk as shown. Determine its angular velocity at the instant its mass center G has moved distance d down along the inclined plane. The disk rolls without slipping.

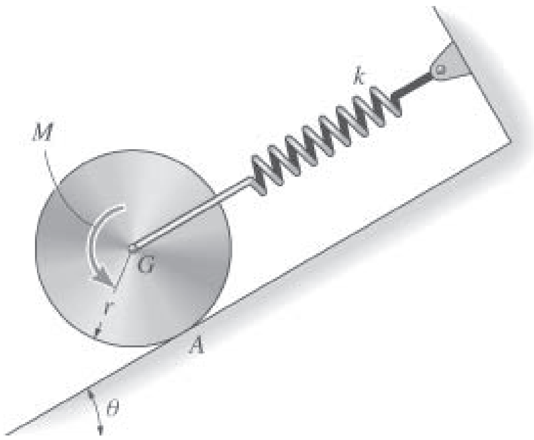
Given:

$m_d = 20\ \text{kg}$ $\theta = 30\ \text{deg}$

$M = 30\ \text{N}\cdot\text{m}$ $r = 0.2\ \text{m}$

$d = 0.8\ \text{m}$ $g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

$k = 150\ \dfrac{\text{N}}{\text{m}}$



Solution: Guess $\omega = 1\ \dfrac{\text{rad}}{\text{s}}$

Initial stretch in the spring $k\,d_0 = m_d\,g\sin(\theta)$

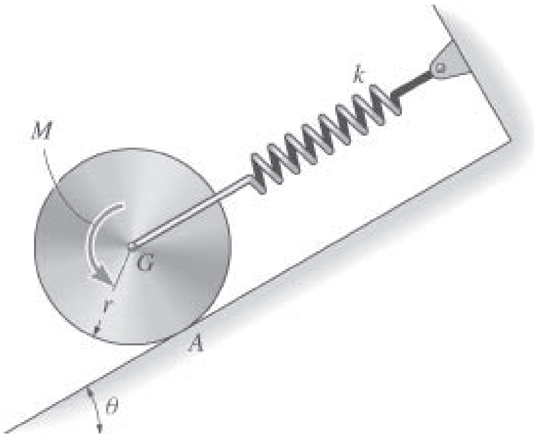
$d_0 = \dfrac{m_d\,g\sin(\theta)}{k}$ $d_0 = 0.654\ \text{m}$

Given

$$M\frac{d}{r} + m_d\,g\,d\sin(\theta) - \frac{k}{2}\left[\left(d + d_0\right)^2 - d_0{}^2\right] = \frac{1}{2}m_d\left(\omega r\right)^2 + \frac{1}{2}\left(\frac{1}{2}m_d\,r^2\right)\omega^2$$

$\omega = \text{Find}(\omega)$ $\qquad \omega = 11.0\,\dfrac{\text{rad}}{\text{s}}$

Problem 18-23



The disk of mass m_d is originally at rest, and the spring holds it in equilibrium. A couple moment M is then applied to the disk as shown. Determine how far the center of mass of the disk travels down along the incline, measured from the equilibrium position, before it stops. The disk rolls without slipping.

Given:

$m_d = 20\ \text{kg}$

$M = 30\ \text{N}\cdot\text{m}$

$k = 150\ \dfrac{\text{N}}{\text{m}}$

$\theta = 30\ \text{deg}$

$r = 0.2\ \text{m}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Solution: Guess $d = 3\ \text{m}$

Initial stretch in the spring $\quad k\,d_0 = m_d\,g\sin(\theta)$

$$d_0 = \frac{m_d\,g\sin(\theta)}{k} \qquad d_0 = 0.654\ \text{m}$$

Given $\quad M\dfrac{d}{r} + m_d\,g\,d\sin(\theta) - \dfrac{k}{2}\left[\left(d+d_0\right)^2 - d_0^{\,2}\right] = 0$

$d = \text{Find}(d) \qquad d = 2\ \text{m}$

***Problem 18-24**

The linkage consists of two rods AB and CD each of weight W_1 and bar AD of weight W_2. When $\theta = 0$, rod AB is rotating with angular velocity ω_O. If rod CD is subjected to a couple moment M

and bar AD is subjected to a horizontal force P as shown, determine ω_{AB} at the instant $\theta = \theta_1$.

Given:

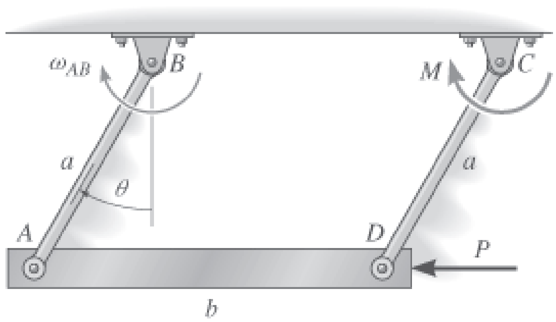
$W_1 = 8\ \text{lb}$ $a = 2\ \text{ft}$

$W_2 = 10\ \text{lb}$ $b = 3\ \text{ft}$

$\omega_0 = 2\ \dfrac{\text{rad}}{\text{s}}$ $\theta_1 = 90\ \text{deg}$

$P = 20\ \text{lb}$ $M = 15\ \text{lb}\cdot\text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution:

$$U = P a \sin(\theta_1) + M\theta_1 - 2W_1 \frac{a}{2}(1 - \cos(\theta_1)) - W_2 a(1 - \cos(\theta_1))$$

Guess $\omega = 1\ \dfrac{\text{rad}}{\text{s}}$

Given

$$\frac{1}{2}2\left(\frac{W_1}{g}\frac{a^2}{3}\right)\omega_0^{\,2} + \frac{1}{2}\left(\frac{W_2}{g}\right)(a\omega_0)^2 + U = \frac{1}{2}2\left(\frac{W_1}{g}\frac{a^2}{3}\right)\omega^2 + \frac{1}{2}\left(\frac{W_2}{g}\right)(a\omega)^2$$

$\omega = \text{Find}(\omega)$ $\omega = 5.739\ \dfrac{\text{rad}}{\text{s}}$

Problem 18-25

The linkage consists of two rods AB and CD each of weight W_1 and bar AD of weight W_2. When $\theta = 0$, rod AB is rotating with angular velocity ω_O. If rod CD is subjected to a couple moment M and bar AD is subjected to a horizontal force P as shown, determine ω_{AB} at the instant $\theta = \theta_1$.

Given:

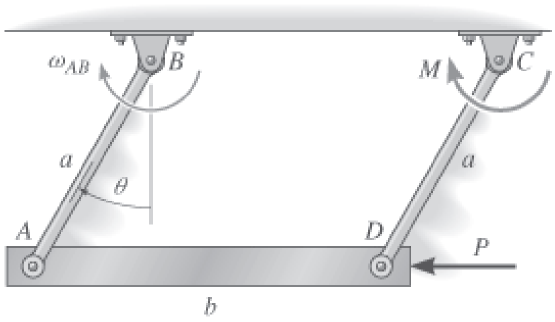
$W_1 = 8\ \text{lb}$ $\quad a = 2\ \text{ft}$

$W_2 = 10\ \text{lb}$ $\quad b = 3\ \text{ft}$

$\omega_0 = 2\ \frac{\text{rad}}{\text{s}}$ $\quad \theta_1 = 45\ \text{deg}$

$M = 15\ \text{lb}\cdot\text{ft}$ $\quad P = 20\ \text{lb}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

$$U = P\,a\sin(\theta_1) + M\theta_1 - 2W_1\frac{a}{2}\left(1-\cos(\theta_1)\right) - W_2\,a\left(1-\cos(\theta_1)\right)$$

Guess $\quad \omega = 1\ \frac{\text{rad}}{\text{s}}$

Given $$\frac{1}{2}2\left(\frac{W_1}{g}\frac{a^2}{3}\right)\omega_0^{\,2} + \frac{1}{2}\left(\frac{W_2}{g}\right)\left(a\omega_0\right)^2 + U = \frac{1}{2}2\left(\frac{W_1}{g}\frac{a^2}{3}\right)\omega^2 + \frac{1}{2}\left(\frac{W_2}{g}\right)(a\omega)^2$$

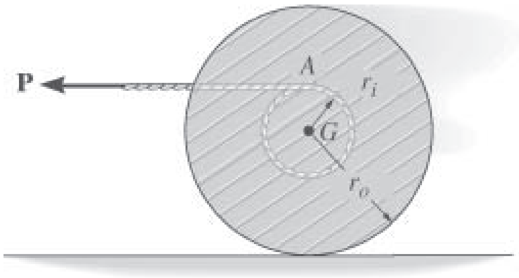
$\omega = \text{Find}(\omega)$ $\quad \omega = 5.916\ \frac{\text{rad}}{\text{s}}$

Problem 18-26

The spool has weight W and radius of gyration k_G. A horizontal force P is applied to a cable wrapped around its inner core. If the spool is originally at rest, determine its angular velocity after the mass center G has moved distance d to the left. The spool rolls without slipping. Neglect the mass of the cable.

Given:

$W = 500\ \text{lb}$ $\quad d = 6\ \text{ft}$



$k_G = 1.75\ \text{ft}$ $\quad r_i = 0.8\ \text{ft}$

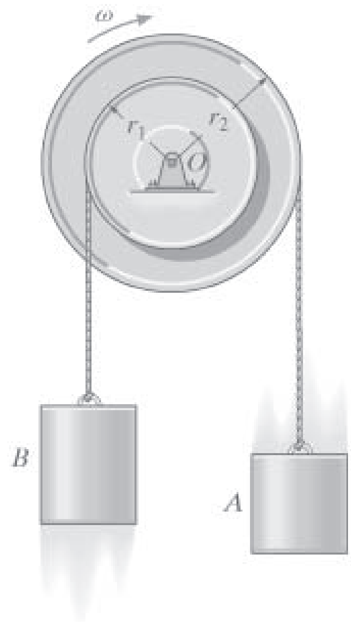
$P = 15\ \text{lb}$ $\quad r_o = 2.4\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

Solution: Guess $\omega = 1\ \dfrac{\text{rad}}{\text{s}}$

Given $\quad P\left(\dfrac{r_o + r_i}{r_o}\right)d = \dfrac{1}{2}\left(\dfrac{W}{g}\right)k_G^{\,2}\,\omega^2 + \dfrac{1}{2}\left(\dfrac{W}{g}\right)(r_o\omega)^2 \qquad \omega = \text{Find}(\omega) \qquad \omega = 1.324\ \dfrac{\text{rad}}{\text{s}}$

Problem 18-27



The double pulley consists of two parts that are attached to one another. It has a weight W_p and a centroidal radius of gyration k_O and is turning with an angular velocity ω clockwise. Determine the kinetic energy of the system. Assume that neither cable slips on the pulley.

Given:

$W_P = 50\ \text{lb}$ $\quad r_1 = 0.5\ \text{ft}$

$W_A = 20\ \text{lb}$ $\quad r_2 = 1\ \text{ft}$

$W_B = 30\ \text{lb}$ $\quad k_O = 0.6\ \text{ft}$

$\omega = 20\ \dfrac{\text{rad}}{\text{s}}$

Solution:

$$K_E = \frac{1}{2}I\omega^2 + \frac{1}{2}W_A v_A^{\,2} + \frac{1}{2}W_B v_B^{\,2}$$

$$K_E = \frac{1}{2}\left(\frac{W_P}{g}\right)k_O^{\,2}\,\omega^2 + \frac{1}{2}\left(\frac{W_A}{g}\right)(r_2\omega)^2 + \frac{1}{2}\left(\frac{W_B}{g}\right)(r_1\omega)^2$$

$K_E = 283\ \text{ft}\cdot\text{lb}$

***Problem 18-28**

The system consists of disk A of weight W_A, slender rod BC of weight W_{BC}, and smooth collar C of weight W_C. If the disk rolls without slipping, determine the velocity of the collar at the instant the rod becomes horizontal, i.e. $\theta = 0°$. The system is released from rest when $\theta = \theta_0$.

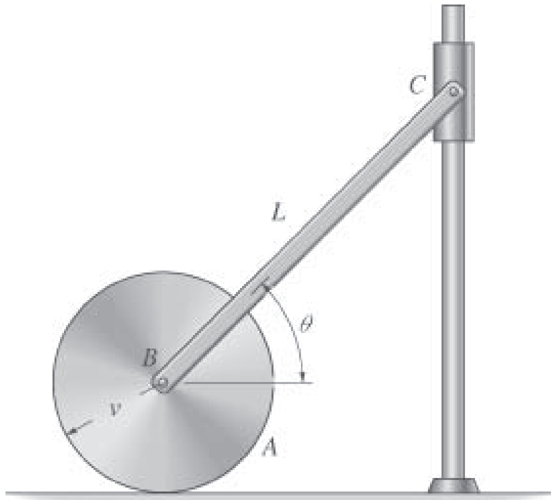
Given:

$W_A = 20 \text{ lb}$ $\quad L = 3 \text{ ft}$

$W_{BC} = 4 \text{ lb}$ $\quad r = 0.8 \text{ ft}$

$W_C = 1 \text{ lb}$ $\quad g = 32.2 \frac{\text{ft}}{\text{s}^2}$

$\theta_0 = 45 \text{ deg}$



Solution:

Guess $\quad v_C = 1 \frac{\text{ft}}{\text{s}}$

Given

$$W_{BC}\frac{L}{2}\cos(\theta_0) + W_C L\cos(\theta_0) = \frac{1}{2}\left(\frac{W_C}{g}\right)v_C^2 + \frac{1}{2}\left(\frac{W_{BC}}{g}\frac{L^2}{3}\right)\left(\frac{v_C}{L}\right)^2$$

$v_C = \text{Find}(v_C)$ $\qquad v_C = 13.3 \frac{\text{ft}}{\text{s}}$

Problem 18-29

The cement bucket of weight W_1 is hoisted using a motor that supplies a torque **M** to the axle of the wheel. If the wheel has a weight W_2 and a radius of gyration about O of k_O, determine the speed of the bucket when it has been hoisted a distance h starting from rest.

Given:

$W_1 = 1500\ \text{lb}$

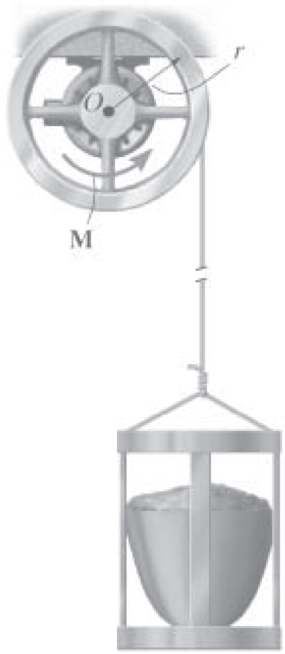
$W_2 = 115\ \text{lb}$

$M = 2000\ \text{lb}\cdot\text{ft}$

$k_O = 0.95\ \text{ft}$

$h = 10\ \text{ft}$

$r = 1.25\ \text{ft}$



Solution:

Guess $\quad v = 1\ \dfrac{\text{ft}}{\text{s}}$

Given $\quad M\dfrac{h}{r} = \dfrac{1}{2}\left(\dfrac{W_1}{g}\right)v^2 + \dfrac{1}{2}\left(\dfrac{W_2}{g}\right)k_O{}^2\left(\dfrac{v}{r}\right)^2 + W_1 h$

$v = \text{Find}(v) \qquad v = 6.41\ \dfrac{\text{ft}}{\text{s}}$

Problem 18-30

The assembly consists of two slender rods each of weight W_r and a disk of weight W_d. If the spring is unstretched when $\theta = \theta_1$ and the assembly is released from rest at this position, determine the angular velocity of rod AB at the instant $\theta = 0$. The disk rolls without slipping.

Given:

$W_r = 15\ \text{lb}$

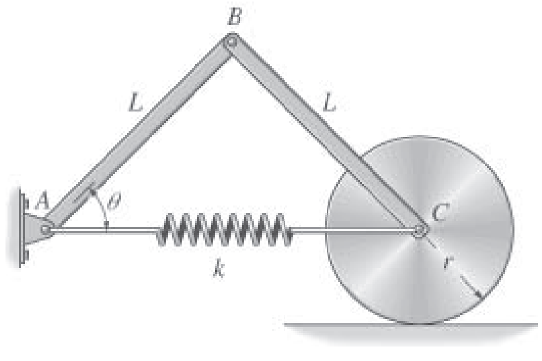
$W_d = 20\ \text{lb}$

$\theta_1 = 45\ \text{deg}$

$k = 4\ \dfrac{\text{lb}}{\text{ft}}$

$L = 3\ \text{ft}$

$r = 1\ \text{ft}$



Solution:

Guess $\quad \omega = 1\ \dfrac{\text{rad}}{\text{s}}$

Given $$2W_r\left(\frac{L}{2}\right)\sin(\theta_1) - \frac{1}{2}k\left(2L - 2L\cos(\theta_1)\right)^2 = 2\frac{1}{2}\left(\frac{1}{3}\frac{W_r}{g}L^2\right)\omega^2$$

$\omega = \text{Find}(\omega)$ $\qquad \omega = 4.284\,\dfrac{\text{rad}}{\text{s}}$

Problem 18-31

The uniform door has mass M and can be treated as a thin plate having the dimensions shown. If it is connected to a torsional spring at A, which has stiffness k, determine the required initial twist of the spring in radians so that the door has an angular velocity ω when it closes at $\theta = 0°$ after being opened at $\theta = 90°$ and released from rest. *Hint:* For a torsional spring $M = k\theta$, where k is the stiffness and θ is the angle of twist.

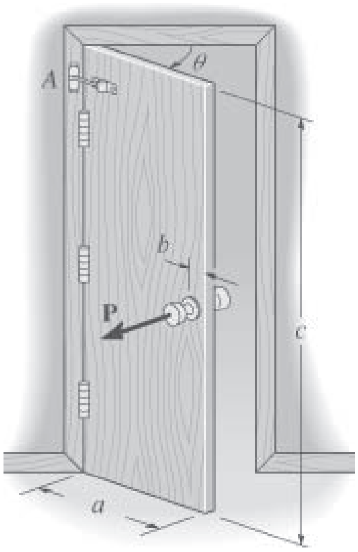
Given:

$M = 20$ kg $\qquad a = 0.8$ m

$k = 80\,\dfrac{\text{N}\cdot\text{m}}{\text{rad}}$ $\qquad b = 0.1$ m

$\omega = 12\,\dfrac{\text{rad}}{\text{s}}$ $\qquad c = 2$ m

$P = 0$ N



Solution:

Guess $\qquad \theta_0 = 1$ rad

Given

$$\int_{\theta_0+90\text{ deg}}^{\theta_0} -k\,\theta\,d\theta = \frac{1}{2}\frac{1}{3}M a^2\omega^2$$

$\theta_0 = \text{Find}(\theta_0)$ $\qquad \theta_0 = 1.659$ rad

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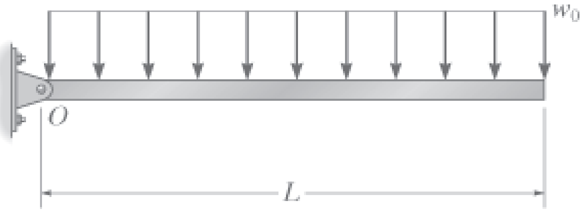
***Problem 18-32**

The uniform slender bar has a mass m and a length L. It is subjected to a uniform distributed load w_0 which is always directed perpendicular to the axis of the bar. If it is released from the position shown, determine its angular velocity at the instant it has rotated 90°. Solve the problem for rotation in (a) the horizontal plane, and (b) the vertical plane.

Solution:



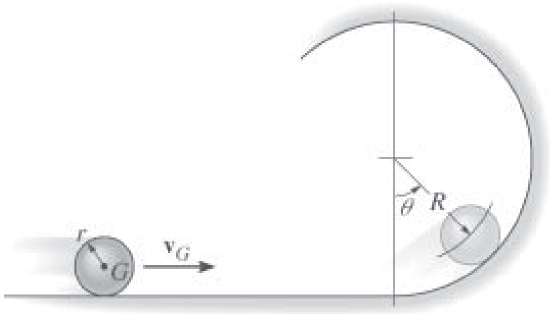
$$\int_0^{\frac{\pi}{2}}\int_0^L x\,w_0\;dx\;d\theta = \frac{1}{4}\pi L^2 w_0$$

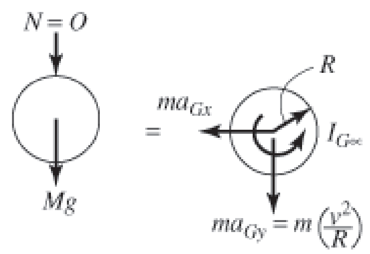
(a) $$\frac{1}{4}\pi L^2 w_0 = \frac{1}{2}\left(\frac{1}{3}m L^2\right)\omega^2$$ $$\omega = \sqrt{\frac{3\pi w_0}{2m}}$$

(b) $$\frac{1}{4}\pi L^2 w_0 = \frac{1}{2}\left(\frac{1}{3}m L^2\right)\omega^2 - m g\frac{L}{2}$$ $$\omega = \sqrt{\frac{3\pi w_0}{2m} + \frac{3g}{L}}$$

Problem 18-33

A ball of mass m and radius r is cast onto the horizontal surface such that it rolls without slipping. Determine the minimum speed v_G of its mass center G so that it rolls completely around the loop of radius $R + r$ without leaving the track.





Solution:

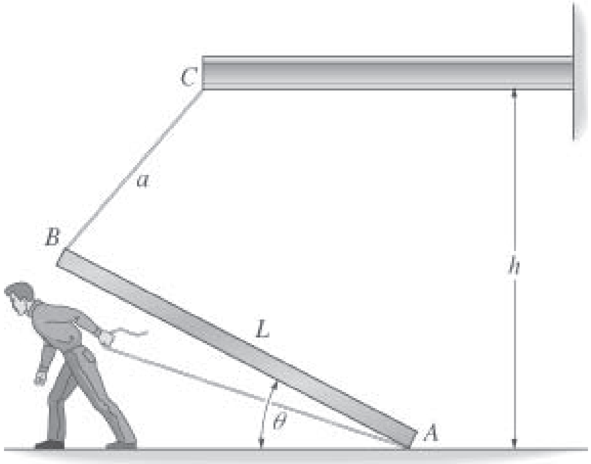
$$m g = m\left(\frac{v^2}{R}\right) \qquad v^2 = g R$$

$$\frac{1}{2}\left(\frac{2}{5}m r^2\right)\left(\frac{v_G}{r}\right)^2 + \frac{1}{2}m v_G^2 - m g 2R = \frac{1}{2}\left(\frac{2}{5}m r^2\right)\left(\frac{v}{r}\right)^2 + \frac{1}{2}m v^2$$

$$\frac{1}{5}v_G^2 + \frac{1}{2}v_G^2 = 2g R + \frac{1}{5}g R + \frac{1}{2}g R$$ $$v_G = 3\sqrt{\frac{3}{7}g R}$$

Problem 18-34

The beam has weight W and is being raised to a vertical position by pulling very slowly on its bottom end A. If the cord fails when $\theta = \theta_1$ and the beam is essentially at rest, determine the speed of A at the instant cord BC becomes vertical. Neglect friction and the mass of the cords, and treat the beam as a slender rod.



Given:

$W = 1500 \text{ lb}$

$\theta_1 = 60 \text{ deg}$

$L = 13 \text{ ft}$

$h = 12 \text{ ft}$

$a = 7 \text{ ft}$

$g = 32.2 \dfrac{\text{ft}}{\text{s}^2}$

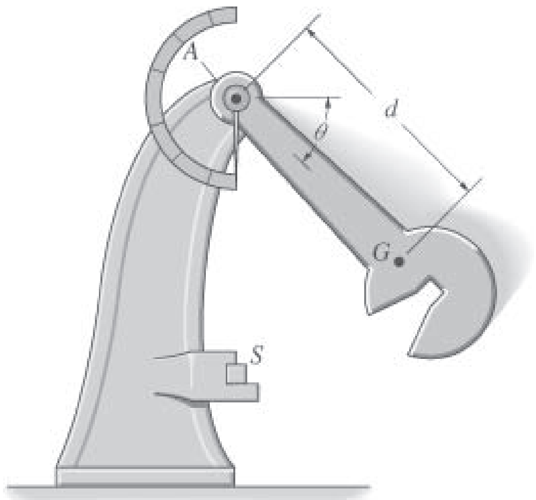
Solution:

$$W\left[\frac{L}{2}\sin(\theta_1) - \left(\frac{h-a}{2}\right)\right] = \frac{1}{2}\left(\frac{W}{g}\right)v_A^2$$

$$v_A = \sqrt{2g\left[\frac{L}{2}\sin(\theta_1) - \left(\frac{h-a}{2}\right)\right]} \qquad v_A = 14.2 \frac{\text{ft}}{\text{s}}$$

Problem 18-35

The pendulum of the Charpy impact machine has mass M and radius of gyration k_A. If it is released from rest when $\theta = 0°$, determine its angular velocity just before it strikes the specimen S, $\theta = 90°$, using the conservation of energy equation.



Given:

$M = 50 \text{ kg}$

$k_A = 1.75 \text{ m}$

$d = 1.25 \text{ m}$

$g = 9.81 \dfrac{\text{m}}{\text{s}^2}$

Solution:

$$0 + M g d = 0 + \frac{1}{2} M k_A^2 \omega_2^2 \qquad \omega_2 = \sqrt{\frac{2 g d}{k_A^2}}$$

$$\omega_2 = 2.83 \frac{\text{rad}}{\text{s}}$$

***Problem 18-36**

The soap-box car has weight W_c including the passenger but *excluding* its four wheels. Each wheel has weight W_w radius *r*, and radius of gyration *k*, computed about an axis passing through the wheel's axle. Determine the car's speed after it has traveled distance *d* starting from rest. The wheels roll without slipping. Neglect air resistance. Solve using conservation of energy.

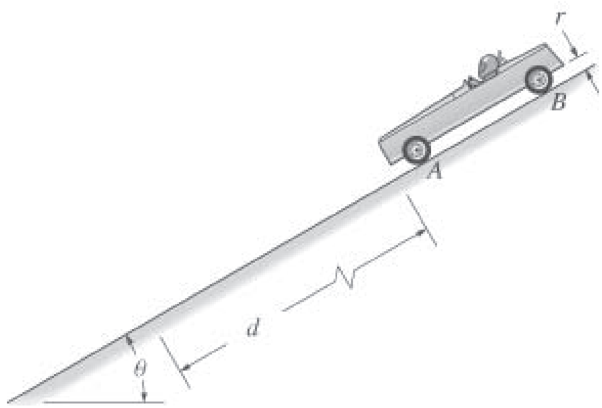
Given:

$W_c = 110 \text{ lb}$ $\quad d = 100 \text{ ft}$

$W_w = 5 \text{ lb}$ $\quad \theta = 30 \text{ deg}$

$r = 0.5 \text{ ft}$ $\quad g = 32.2 \frac{\text{ft}}{\text{s}^2}$

$k = 0.3 \text{ ft}$



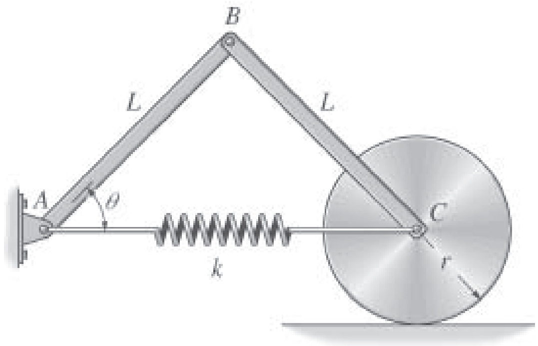
Solution:

$$0 + (W_c + 4W_w) d \sin(\theta) = 0 + \frac{1}{2}\left(\frac{W_c + 4W_w}{g}\right) v^2 + \frac{1}{2} 4\left(\frac{W_w}{g} k^2\right)\left(\frac{v}{r}\right)^2$$

$$v = \sqrt{\frac{2(W_c + 4W_w) d \sin(\theta) g}{W_c + 4W_w + 4W_w\left(\frac{k^2}{r^2}\right)}} \qquad v = 55.2 \frac{\text{ft}}{\text{s}}$$

Problem 18-37

The assembly consists of two slender rods each of weight W_r and a disk of weight W_d. If the spring is unstretched when $\theta = \theta_1$ and the assembly is released from rest at this position, determine the angular velocity of rod *AB* at the instant $\theta = 0$. The disk rolls without slipping. Solve using the conservation of energy.



Given:

$W_r = 15 \text{ lb} \qquad k = 4 \frac{\text{lb}}{\text{ft}}$

$W_d = 20 \text{ lb} \qquad L = 3 \text{ ft}$

$\theta_1 = 45 \text{ deg} \qquad r = 1 \text{ ft}$

Solution: Guess $\omega = 1 \frac{\text{rad}}{\text{s}}$

Given

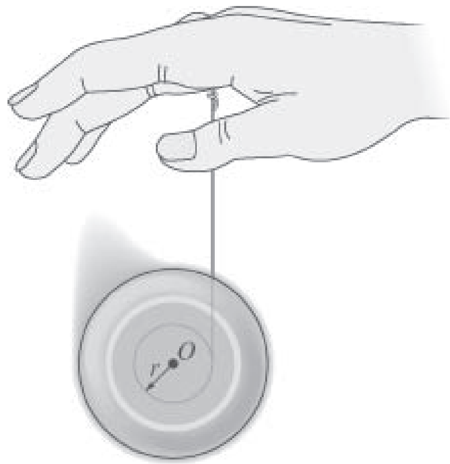
$$0 + 2W_r\left(\frac{L}{2}\right)\sin(\theta_1) = 2\frac{1}{2}\left[\frac{1}{3}\left(\frac{W_r}{g}\right)L^2\right]\omega^2 + \frac{1}{2}k\left(2L - 2L\cos(\theta_1)\right)^2$$

$$\omega = \text{Find}(\omega) \qquad \omega = 4.284 \frac{\text{rad}}{\text{s}}$$

Problem 18-38

A yo-yo has weight W and radius of gyration k_O. If it is released from rest, determine how far it must descend in order to attain angular velocity ω. Neglect the mass of the string and assume that the string is wound around the central peg such that the mean radius at which it unravels is r. Solve using the conservation of energy.

Given:



$W = 0.3 \text{ lb}$

$k_O = 0.06 \text{ ft}$

$\omega = 70 \frac{\text{rad}}{\text{s}}$

$r = 0.02 \text{ ft}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$

Solution:

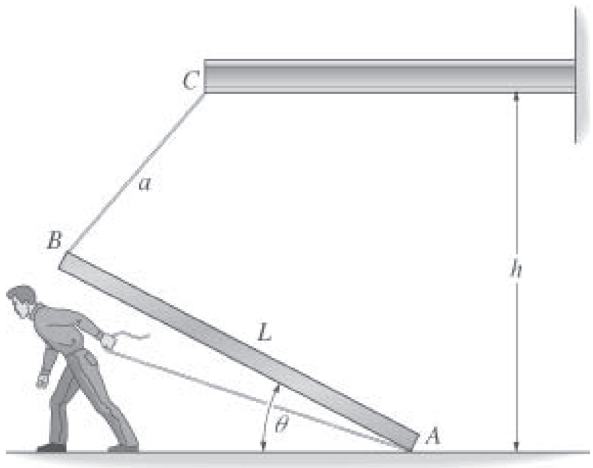
$$0 + W s = \frac{1}{2}\left(\frac{W}{g}\right)(r\omega)^2 + \frac{1}{2}\left(\frac{W}{g}k_O^2\right)\omega^2 + 0$$

$$s = \left(\frac{r^2 + k_O^2}{2g}\right)\omega^2 \qquad s = 0.304 \text{ ft}$$

Problem 18-39

The beam has weight W and is being raised to a vertical position by pulling very slowly on its bottom end A. If the cord fails when $\theta = \theta_1$ and the beam is essentially at rest, determine the speed of A at the instant cord BC becomes vertical. Neglect friction and the mass of the cords, and treat the beam as a slender rod. Solve using the conservation of energy.

Given:



$W = 1500\ \text{lb}$

$\theta_1 = 60\ \text{deg}$

$L = 13\ \text{ft}$

$h = 12\ \text{ft}$

$a = 7\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

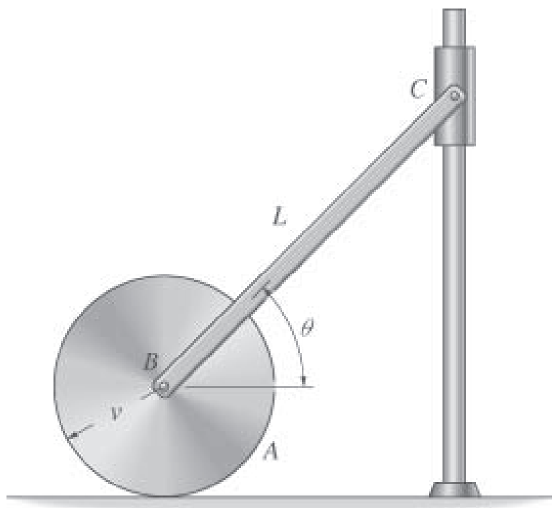
Solution:

$$0 + W\left(\frac{L}{2}\right)\sin(\theta_1) = \frac{1}{2}\left(\frac{W}{g}\right)v_A^2 + W\left(\frac{h-a}{2}\right)$$

$$v_A = \sqrt{2g\left[\frac{L}{2}\sin(\theta_1) - \left(\frac{h-a}{2}\right)\right]} \qquad v_A = 14.2\ \frac{\text{ft}}{\text{s}}$$

***Problem 18-40**

The system consists of disk A of weight W_A, slender rod BC of weight W_{BC}, and smooth collar C of weight W_C. If the disk rolls without slipping, determine the velocity of the collar at the instant the rod becomes horizontal, i.e. $\theta = 0°$. The system is released from rest when $\theta = \theta_0$. Solve using the conservation of energy.



Given:

$W_A = 20\ \text{lb}$ $\quad L = 3\ \text{ft}$

$W_{BC} = 4\ \text{lb}$ $\quad r = 0.8\ \text{ft}$

$W_C = 1\ \text{lb} \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$\theta_0 = 45\ \text{deg}$

Solution:

Guess $\quad v_C = 1\ \frac{\text{ft}}{\text{s}}$

Given

$$0 + W_{BC}\left(\frac{L}{2}\right)\cos(\theta_0) + W_C L\cos(\theta_0) = \frac{1}{2}\left(\frac{W_C}{g}\right)v_C^2 + \frac{1}{2}\left(\frac{W_{BC}}{g}\frac{L^2}{3}\right)\left(\frac{v_C}{L}\right)^2 + 0$$

$v_C = \text{Find}(v_C) \qquad v_C = 13.3\ \frac{\text{ft}}{\text{s}}$

Problem 18-41

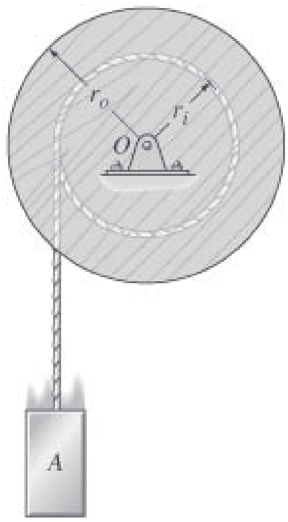
The spool has mass m_S and radius of gyration k_O. If block A of mass m_A is released from rest, determine the distance the block must fall in order for the spool to have angular velocity ω. Also, what is the tension in the cord while the block is in motion? Neglect the mass of the cord.

Given:

$m_S = 50\ \text{kg} \qquad r_i = 0.2\ \text{m} \qquad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$m_A = 20\ \text{kg} \qquad r_o = 0.3\ \text{m}$

$\omega = 5\ \frac{\text{rad}}{\text{s}} \qquad k_O = 0.280\ \text{m}$



Solution:

Guesses $\quad d = 1\ \text{m} \quad T = 1\ \text{N}$

Given

$$0 + 0 = \frac{1}{2}m_S k_O^2\,\omega^2 + \frac{1}{2}m_A(r_i\omega)^2 - m_A g d$$

$$0 + 0 - Td = \frac{1}{2}m_A(r_i\omega)^2 - m_A g d$$

$\begin{pmatrix} d \\ T \end{pmatrix} = \text{Find}(d, T) \qquad d = 0.301\ \text{m} \qquad T = 163\ \text{N}$

Problem 18-42

When slender bar *AB* of mass *M* is horizontal it is at rest and the spring is unstretched. Determine the stiffness *k* of the spring so that the motion of the bar is momentarily stopped when it has rotated downward 90°.

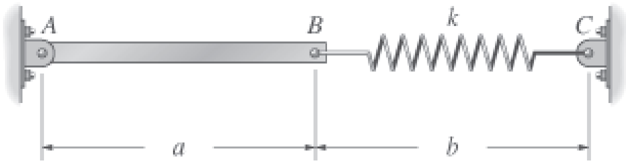
Given:

$M = 10\ \text{kg}$

$a = 1.5\ \text{m}$

$b = 1.5\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution:

$$0 + 0 = 0 + \frac{1}{2}k\left[\sqrt{(a+b)^2 + a^2} - b\right]^2 - Mg\frac{a}{2}$$

$$k = \frac{Mga}{\left[\sqrt{(a+b)^2 + a^2} - b\right]^2} \qquad k = 42.8\ \frac{\text{N}}{\text{m}}$$

Problem 18-43

The disk of weight *W* is rotating about pin *A* in the vertical plane with an angular velocity ω_1 when $\theta = 0°$. Determine its angular velocity at the instant shown, $\theta = 90$ deg. Also, compute the horizontal and vertical components of reaction at *A* at this instant.

Given:

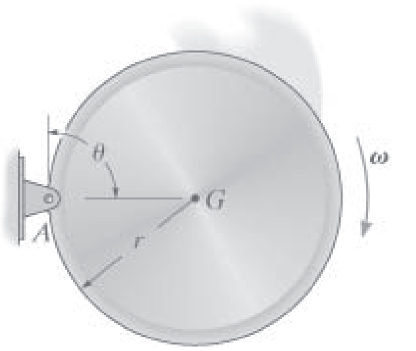
$W = 15\ \text{lb}$

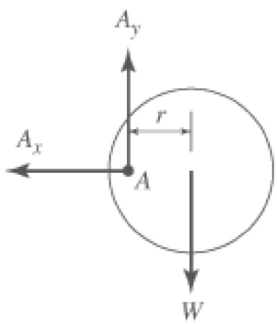
$\omega_1 = 2\ \frac{\text{rad}}{\text{s}}$

$\theta = 90\ \text{deg}$

$r = 0.5\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$





Solution:

Guesses $\quad A_x = 1\ \text{lb} \quad A_y = 1\ \text{lb} \quad \omega_2 = 1\ \frac{\text{rad}}{\text{s}} \quad \alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

Given $\frac{1}{2}\left(\frac{3}{2}\frac{W}{g}r^2\right)\omega_1{}^2 + W r = \frac{1}{2}\left(\frac{3}{2}\frac{W}{g}r^2\right)\omega_2{}^2$

$$-A_x = \left(\frac{-W}{g}\right)r\omega_2{}^2 \qquad A_y - W = \left(\frac{-W}{g}\right)\alpha r \qquad -W r = \frac{-3}{2}\left(\frac{W}{g}\right)r^2\alpha$$

$$\begin{pmatrix} A_x \\ A_y \\ \omega_2 \\ \alpha \end{pmatrix} = \operatorname{Find}(A_x, A_y, \omega_2, \alpha) \qquad \alpha = 42.9\ \frac{\text{rad}}{\text{s}^2} \qquad \omega_2 = 9.48\ \frac{\text{rad}}{\text{s}} \qquad \begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 20.9 \\ 5.0 \end{pmatrix} \text{lb}$$

***Problem 18-44**

The door is made from one piece, whose ends move along the horizontal and vertical tracks. If the door is in the open position $\theta = 0°$, and then released, determine the speed at which its end A strikes the stop at C. Assume the door is a thin plate of weight W having width c.

Given:

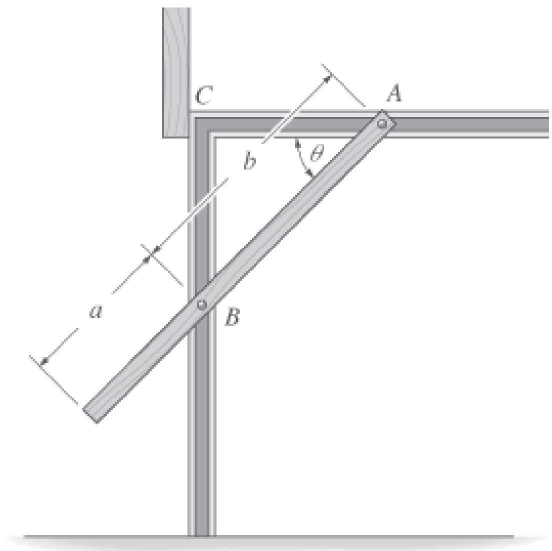
$W = 180$ lb

$a = 3$ ft

$b = 5$ ft

$c = 10$ ft

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

Guesses $v_A = 1\ \frac{\text{ft}}{\text{s}}$ $\omega = 1\ \frac{\text{rad}}{\text{s}}$

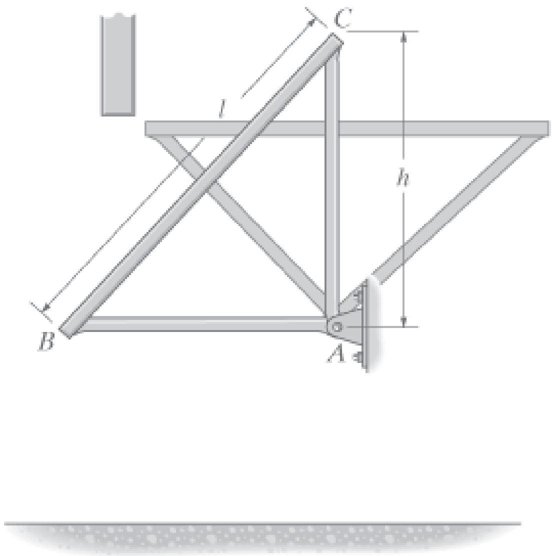
Given

$$0 + 0 = \frac{1}{2}\frac{W}{g}\left[\frac{(a+b)^2}{12} + \left(b - \frac{a+b}{2}\right)^2\right]\omega^2 - W\left(\frac{a+b}{2}\right) \qquad v_A = \omega b$$

$$\begin{pmatrix} \omega \\ v_A \end{pmatrix} = \operatorname{Find}(\omega, v_A) \qquad \omega = 6.378\ \frac{\text{rad}}{\text{s}} \qquad v_A = 31.9\ \frac{\text{ft}}{\text{s}}$$

Problem 18-45

The overhead door BC is pushed slightly from its open position and then rotates downward about the pin at A. Determine its angular velocity just before its end B strikes the floor. Assume the door is a thin plate having a mass M and length l. Neglect the mass of the supporting frame AB and AC.



Given:

$M = 180 \text{ kg}$

$l = 6 \text{ m}$

$h = 5 \text{ m}$

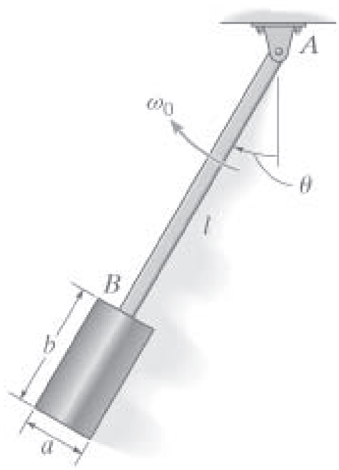
Solution: $d = \sqrt{h^2 - \left(\frac{l}{2}\right)^2}$

Guess $\omega = 1 \frac{\text{rad}}{\text{s}}$

Given $M g d = \frac{1}{2}\left(\frac{M l^2}{12} + M d^2\right)\omega^2$ $\omega = \text{Find}(\omega)$ $\omega = 2.03 \frac{\text{rad}}{\text{s}}$

Problem 18-46

The cylinder of weight W_1 is attached to the slender rod of weight W_2 which is pinned at point A. At the instant $\theta = \theta_0$ the rod has an angular velocity ω_0 as shown. Determine the angle θ_f to which the rod swings before it momentarily stops.



Given:

$W_1 = 80 \text{ lb}$ $a = 1 \text{ ft}$

$W_2 = 10 \text{ lb}$ $b = 2 \text{ ft}$

$\omega_0 = 1 \frac{\text{rad}}{\text{s}}$ $l = 5 \text{ ft}$

$\theta_0 = 30 \text{ deg}$ $g = 32.2 \frac{\text{ft}}{\text{s}^2}$

Solution:

$$I_A = \frac{W_1}{g}\left[\frac{1}{4}\left(\frac{a}{2}\right)^2 + \frac{b^2}{12}\right] + \frac{W_1}{g}\left(l + \frac{b}{2}\right)^2 + \left(\frac{W_2}{g}\right)\frac{l^2}{3}$$

$$d = \frac{W_1\left(l + \frac{b}{2}\right) + W_2\left(\frac{l}{2}\right)}{W_1 + W_2}$$

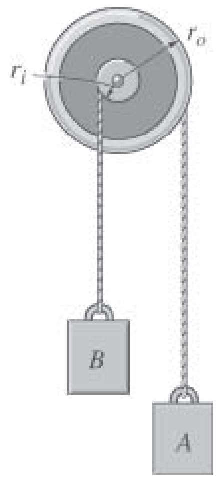
Guess $\theta_f = 1 \text{ deg}$

Given $\frac{1}{2}I_A\omega_0^2 - (W_1 + W_2)d\cos(\theta_0) = -(W_1 + W_2)d\cos(\theta_f)$

$\theta_f = \text{Find}(\theta_f)$ $\theta_f = 39.3 \text{ deg}$

Problem 18-47

The compound disk pulley consists of a hub and attached outer rim. If it has mass m_P and radius of gyration k_G, determine the speed of block A after A descends distance d from rest. Blocks A and B each have a mass m_b. Neglect the mass of the cords.



Given:

$m_p = 3 \text{ kg}$ $r_i = 30 \text{ mm}$ $m_b = 2 \text{ kg}$

$k_G = 45 \text{ mm}$ $r_o = 100 \text{ mm}$

$d = 0.2 \text{ m}$ $g = 9.81 \frac{\text{m}}{\text{s}^2}$

Solution:

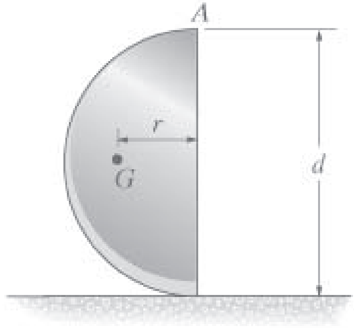
Guess $v_A = 1 \frac{\text{m}}{\text{s}}$

Given

$$0 + 0 = \frac{1}{2}m_b v_A^2 + \frac{1}{2}m_b\left(\frac{r_i}{r_o}v_A\right)^2 + \frac{1}{2}\left(m_p k_G^2\right)\left(\frac{v_A}{r_o}\right)^2 - m_b g d + m_b g\left(\frac{r_i}{r_o}\right)d$$

$v_A = \text{Find}(v_A)$ $v_A = 1.404 \frac{\text{m}}{\text{s}}$

***Problem 18-48**



The semicircular segment of mass M is released from rest in the position shown. Determine the velocity of point A when it has rotated counterclockwise 90°. Assume that the segment rolls without slipping on the surface. The moment of inertia about its mass center is I_G.

Given:

$M = 15\ \text{kg}$ $\quad r = 0.15\ \text{m}$

$I_G = 0.25\ \text{kg}\cdot\text{m}^2$ $\quad d = 0.4\ \text{m}$

Solution:

Guesses $\quad \omega = 1\ \dfrac{\text{rad}}{\text{s}} \quad v_G = 1\ \dfrac{\text{m}}{\text{s}}$

Given $\quad M g d = \dfrac{1}{2} M v_G^2 + \dfrac{1}{2} I_G \omega^2 + M g(d - r) \qquad v_G = \omega\left(\dfrac{d}{2} - r\right)$

$$\begin{pmatrix} \omega \\ v_G \end{pmatrix} = \text{Find}(\omega, v_G) \qquad \omega = 12.4\ \frac{\text{rad}}{\text{s}} \qquad v_G = 0.62\ \frac{\text{m}}{\text{s}}$$

$$\mathbf{v_A} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} \dfrac{-d}{2} \\ \dfrac{d}{2} \\ 0 \end{pmatrix} \qquad \mathbf{v_A} = \begin{pmatrix} -2.48 \\ -2.48 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad \left|\mathbf{v_A}\right| = 3.50\ \frac{\text{m}}{\text{s}}$$

Problem 18-49

The uniform stone (rectangular block) of weight W is being turned over on its side by pulling the vertical cable *slowly* upward until the stone begins to tip. If it then falls freely (**T** = 0) from an essentially balanced at-rest position, determine the speed at which the corner A strikes the pad at B. The stone does not slip at its corner C as it falls.

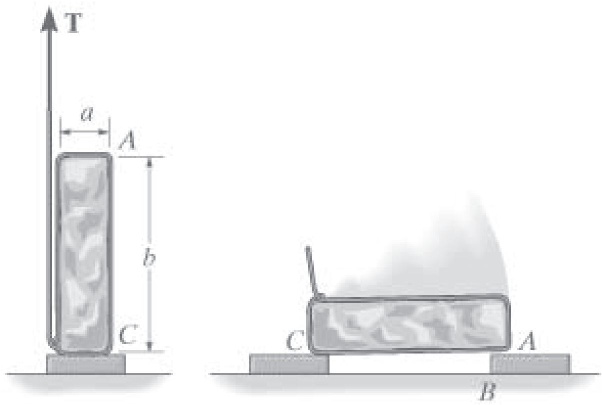
Given:

$W = 150\ \text{lb}$

$a = 0.5\ \text{ft}$

$b = 2\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution:

Guess $\quad \omega = 1\ \dfrac{\text{rad}}{\text{s}}$

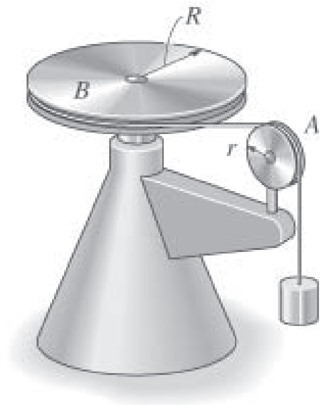
Given $\quad W\left(\frac{\sqrt{a^2+b^2}}{2}\right) = \frac{1}{2}\frac{W}{g}\left(\frac{a^2+b^2}{3}\right)\omega^2 + W\frac{a}{2} \qquad \omega = \text{Find}(\omega)$

$v_A = \omega b \qquad v_A = 11.9\ \frac{\text{ft}}{\text{s}}$

Problem 18-50

The assembly consists of pulley A of mass m_A and pulley B of mass m_B. If a block of mass m_b is suspended from the cord, determine the block's speed after it descends a distance d starting from rest. Neglect the mass of the cord and treat the pulleys as thin disks. No slipping occurs.

Given:



$m_A = 3\ \text{kg}$

$m_B = 10\ \text{kg}$

$m_b = 2\ \text{kg}$

$d = 0.5\ \text{m}$

$r = 30\ \text{mm}$

$R = 100\ \text{mm}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution: Guess $\quad v_b = 1\ \frac{\text{m}}{\text{s}}$

Given

$$0 + 0 = \frac{1}{2}\left(\frac{m_A r^2}{2}\right)\left(\frac{v_b}{r}\right)^2 + \frac{1}{2}\left(\frac{m_B R^2}{2}\right)\left(\frac{v_b}{R}\right)^2 + \frac{1}{2}m_b v_b^2 - m_b g d$$

$v_b = \text{Find}(v_b) \qquad v_b = 1.519\ \frac{\text{m}}{\text{s}}$

Problem 18-51

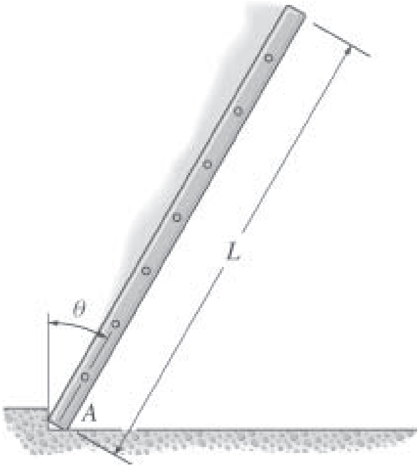
A uniform ladder having weight W is released from rest when it is in the vertical position. If it is allowed to fall freely, determine the angle at which the bottom end A starts to lift off the ground. For the calculation, assume the ladder to be a slender rod and neglect friction at A.

Given:

$W = 30 \text{ lb}$

$L = 10 \text{ ft}$

$g = 32.2 \dfrac{\text{ft}}{\text{s}^2}$



Solution:

(a) The rod will rotate around point A until it loses contact with the horizontal constraint ($A_x = 0$). We will find this point first

Guesses

$\theta_1 = 30 \text{ deg} \quad \omega_1 = 1 \dfrac{\text{rad}}{\text{s}} \qquad \alpha_1 = 1 \dfrac{\text{rad}}{\text{s}^2}$

Given

$$0 + W\left(\frac{L}{2}\right) = \frac{1}{2}\left[\frac{1}{3}\left(\frac{W}{g}\right)L^2\right]\omega_1^{\,2} + W\left(\frac{L}{2}\right)\cos(\theta_1)$$

$$W\left(\frac{L}{2}\right)\sin(\theta_1) = \left[\frac{1}{3}\left(\frac{W}{g}\right)L^2\right]\alpha_1$$

$$\alpha_1\left(\frac{L}{2}\right)\cos(\theta_1) - \omega_1^{\,2}\left(\frac{L}{2}\right)\sin(\theta_1) = 0$$

$$\begin{pmatrix} \omega_1 \\ \alpha_1 \\ \theta_1 \end{pmatrix} = \text{Find}(\omega_1, \alpha_1, \theta_1) \qquad \theta_1 = 48.19 \text{ deg} \qquad \omega_1 = 1.794 \frac{\text{rad}}{\text{s}} \qquad \alpha_1 = 3.6 \frac{\text{rad}}{\text{s}^2}$$

(b) Now the rod moves without any horizontal constraint. If we look for the point at which it loses contact with the floor ($A_y = 0$) we will find that this condition never occurs.

***Problem 18-52**

The slender rod *AB* of weight *W* is attached to a spring *BC* which, has unstretched length *L*. If the rod is released from rest when $\theta = \theta_1$,determine its angular velocity at the instant $\theta = \theta_2$.

Given:

$W = 25 \text{ lb}$

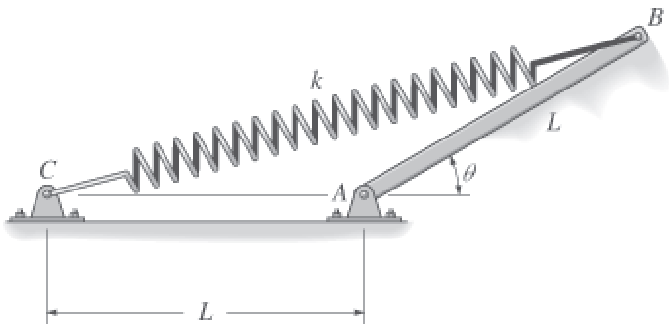
$L = 4\ \text{ft}$

$k = 5\ \dfrac{\text{lb}}{\text{ft}}$

$\theta_1 = 30\ \text{deg}$

$\theta_2 = 90\ \text{deg}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution:

Guess $\omega = 1\ \dfrac{\text{rad}}{\text{s}}$

Given

$$0 + W\left(\frac{L}{2}\right)\sin(\theta_1) + \frac{1}{2}kL^2\left[\sqrt{2(1+\cos(\theta_1))} - 1\right]^2 = \frac{1}{2}\left(\frac{W}{g}\right)\left(\frac{L^2}{3}\right)\omega^2 + W\left(\frac{L}{2}\right)\sin(\theta_2) \ldots$$
$$+ \frac{1}{2}kL^2\left[\sqrt{2(1+\cos(\theta_2))} - 1\right]^2$$

$\omega = \text{Find}(\omega)$ $\quad \omega = 1.178\ \dfrac{\text{rad}}{\text{s}}$

Problem 18-53

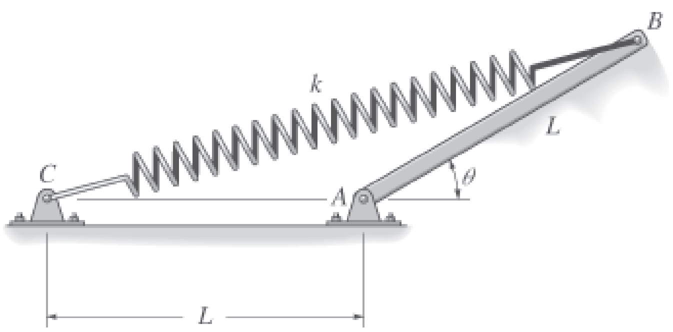
The slender rod AB of weight w is attached to a spring BC which has an unstretched length L. If the rod is released from rest when $\theta = \theta_1$, determine the angular velocity of the rod the instant the spring becomes unstretched.

Given:

$W = 25\ \text{lb}$ $\quad \theta_1 = 30\ \text{deg}$

$L = 4\ \text{ft}$ $\quad g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

$k = 5\ \dfrac{\text{lb}}{\text{ft}}$



Solution:

When the spring is unstretched $\quad \theta_2 = 120\ \text{deg}$

Guess $\omega = 1\ \dfrac{\text{rad}}{\text{s}}$

Given

$$0 + W\left(\frac{L}{2}\right)\sin(\theta_1) + \frac{1}{2}kL^2\left[\sqrt{2(1+\cos(\theta_1))} - 1\right]^2 = \frac{1}{2}\left(\frac{W}{g}\right)\left(\frac{L^2}{3}\right)\omega^2 + W\left(\frac{L}{2}\right)\sin(\theta_2) \ ...$$
$$+ \frac{1}{2}kL^2\left[\sqrt{2(1+\cos(\theta_2))} - 1\right]^2$$

$\omega = \text{Find}(\omega)$ $\quad \omega = 2.817\ \frac{\text{rad}}{\text{s}}$

Problem 18-54

A chain that has a negligible mass is draped over the sprocket which has mass m_s and radius of gyration k_O. If block A of mass m_A is released from rest in the position $s = s_1$, determine the angular velocity of the sprocket at the instant $s = s_2$.

Given:

$m_s = 2\ \text{kg}$

$k_O = 50\ \text{mm}$

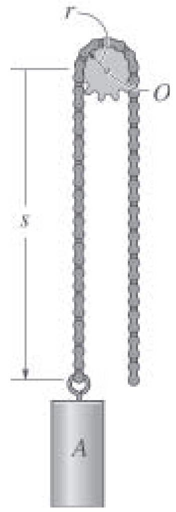
$m_A = 4\ \text{kg}$

$s_1 = 1\ \text{m}$

$s_2 = 2\ \text{m}$

$r = 0.1\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution:

Guess $\quad \omega = 1\ \frac{\text{rad}}{\text{s}}$

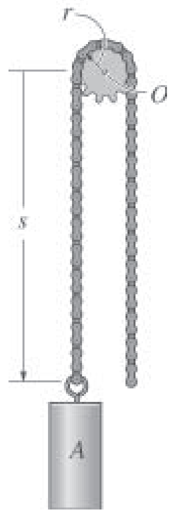
Given

$$0 - m_A g s_1 = \frac{1}{2}m_A(r\omega)^2 + \frac{1}{2}m_s k_O^2\,\omega^2 - m_A g s_2$$

$\omega = \text{Find}(\omega)$ $\quad \omega = 41.8\ \frac{\text{rad}}{\text{s}}$

Problem 18-55

A chain that has a mass density ρ is draped over the sprocket which has mass m_s and radius of gyration k_O. If block A of mass m_A is released from rest in the position $s = s_1$, determine the angular velocity of the sprocket at the instant $s = s_2$. When released there is an equal amount of chain on each side. Neglect the portion of the chain that wraps over the sprocket.



Given:

$m_s = 2\ \text{kg}$ $\quad s_1 = 1\ \text{m}$

$k_O = 50\ \text{mm}$ $\quad s_2 = 2\ \text{m}$

$m_A = 4\ \text{kg}$ $\quad r = 0.1\ \text{m}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$ $\quad \rho = 0.8\ \dfrac{\text{kg}}{\text{m}}$

Solution:

Guess $\quad T_1 = 1\ \text{N m}$ $\quad T_2 = 1\ \text{N m}$ $\quad \omega = 10\ \dfrac{\text{rad}}{\text{s}}$

$V_1 = 1\ \text{N m}$ $\quad V_2 = 1\ \text{N m}$

Given

$$T_1 = 0$$

$$V_1 = -m_A g s_1 - 2\rho s_1 g\left(\frac{s_1}{2}\right)$$

$$T_2 = \frac{1}{2}m_A(r\omega)^2 + \frac{1}{2}m_s k_O^{\,2}\,\omega^2 + \frac{1}{2}\rho(2s_1)(r\omega)^2$$

$$V_2 = -m_A g s_2 - \rho s_2 g\left(\frac{s_2}{2}\right) - \rho(2s_1 - s_2)g\left(\frac{2s_1 - s_2}{2}\right)$$

$$T_1 + V_1 = T_2 + V_2$$

$$\begin{pmatrix} T_1 \\ V_1 \\ T_2 \\ V_2 \\ \omega \end{pmatrix} = \text{Find}(T_1, V_1, T_2, V_2, \omega) \qquad \omega = 39.3\ \frac{\text{rad}}{\text{s}}$$

***Problem 18-56**

Pulley A has weight W_A and centroidal radius of gyration k_B. Determine the speed of the crate C of weight W_C at the instant $s = s_2$. Initially, the crate is released from rest when $s = s_1$. The pulley at P "rolls" downward on the cord without slipping. For the calculation, neglect the mass of this pulley and the cord as it unwinds from the inner and outer hubs of pulley A.

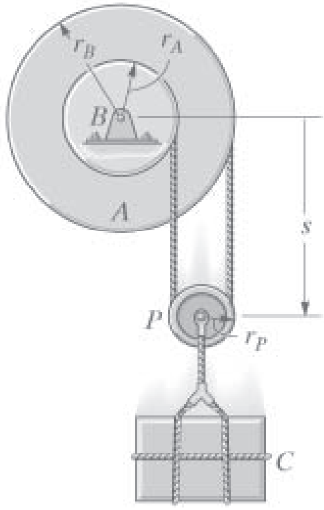
Given:

$W_A = 30\ \text{lb}$ $\quad r_A = 0.4\ \text{ft}$

$W_C = 20\ \text{lb}$ $\quad r_B = 0.8\ \text{ft}$

$k_B = 0.6\ \text{ft}$ $\quad r_P = \dfrac{r_B - r_A}{2}$

$s_1 = 5\ \text{ft}$ $\quad s_2 = 10\ \text{ft}$



Solution:

Guess $\quad \omega = 1\ \dfrac{\text{rad}}{\text{s}} \quad v_C = 1\ \dfrac{\text{ft}}{\text{s}}$

Given

$$-W_C s_1 = \frac{1}{2}\left(\frac{W_A}{g}\right)k_B^2\,\omega^2 + \frac{1}{2}\left(\frac{W_C}{g}\right)v_C^2 - W_C s_2 \qquad v_C = \omega\left(\frac{r_A + r_B}{2}\right)$$

$$\begin{pmatrix}\omega \\ v_C\end{pmatrix} = \text{Find}(\omega, v_C) \qquad \omega = 18.9\ \frac{\text{rad}}{\text{s}} \qquad v_C = 11.3\ \frac{\text{ft}}{\text{s}}$$

Problem 18-57

The assembly consists of two bars of weight W_1 which are pinconnected to the two disks of weight W_2. If the bars are released from rest at $\theta = \theta_0$, determine their angular velocities at the instant $\theta = 0°$. Assume the disks roll without slipping.

Given:

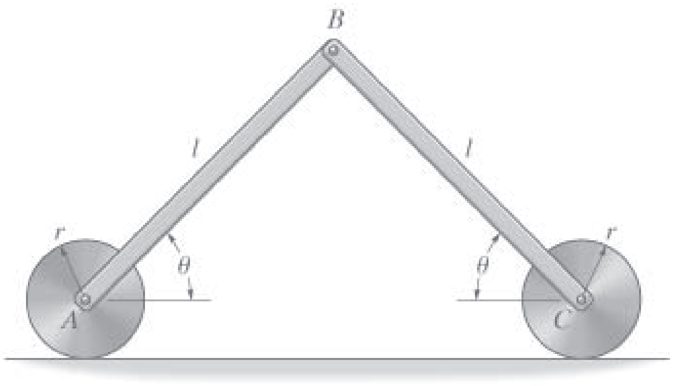
$W_1 = 8\ \text{lb}$

$W_2 = 10\ \text{lb}$

$r = 0.5\ \text{ft}$

$l = 3\ \text{ft}$

$\theta_0 = 60\ \text{deg}$



Solution:

Guess $\omega = 1\ \frac{\text{rad}}{\text{s}}$

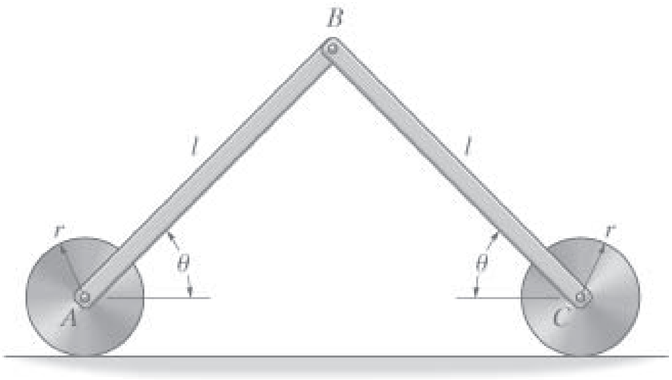
Given $2W_1\left(\frac{l}{2}\right)\sin(\theta_0) = 2\frac{1}{2}\left(\frac{W_1}{g}\right)\left(\frac{l^2}{3}\right)\omega^2$

$\omega = \text{Find}(\omega)$ $\omega = 5.28\ \frac{\text{rad}}{\text{s}}$

Problem 18-58

The assembly consists of two bars of weight W_1 which are pin-connected to the two disks of weight W_2. If the bars are released from rest at $\theta = \theta_1$, determine their angular velocities at the instant $\theta = \theta_2$. Assume the disks roll without slipping.

Given:



$W_1 = 8\ \text{lb}$

$W_2 = 10\ \text{lb}$

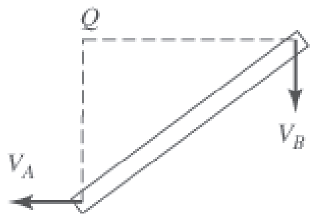
$\theta_1 = 60\ \text{deg}$

$\theta_2 = 30\ \text{deg}$

$r = 0.5\ \text{ft}$

$l = 3\ \text{ft}$

Solution:



Guesses

$\omega = 1\ \frac{\text{rad}}{\text{s}}$ $v_A = 1\ \frac{\text{ft}}{\text{s}}$

Given

$$2W_1\left(\frac{l}{2}\right)\sin(\theta_1) = 2W_1\left(\frac{l}{2}\right)\sin(\theta_2) + 2\frac{1}{2}\left(\frac{W_1}{g}\right)\left(\frac{l^2}{3}\right)\omega^2 + 2\frac{1}{2}\left[\frac{3}{2}\left(\frac{W_2}{g}\right)r^2\right]\left(\frac{v_A}{r}\right)^2$$

$v_A = \omega l \sin(\theta_2)$

$$\begin{pmatrix} \omega \\ v_A \end{pmatrix} = \text{Find}(\omega, v_A) \qquad v_A = 3.32\,\frac{\text{ft}}{\text{s}} \qquad \omega = 2.21\,\frac{\text{rad}}{\text{s}}$$

Problem 18-59

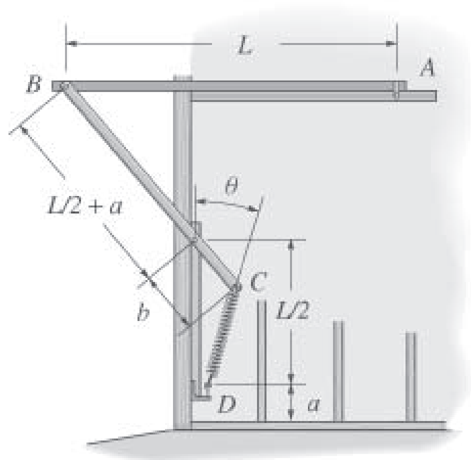
The end A of the garage door AB travels along the horizontal track, and the end of member BC is attached to a spring at C. If the spring is originally unstretched, determine the stiffness k so that when the door falls downward from rest in the position shown, it will have zero angular velocity the moment it closes, i.e., when it and BC become vertical. Neglect the mass of member BC and assume the door is a thin plate having weight W and a width and height of length L. There is a similar connection and spring on the other side of the door.

Given:

$W = 200\text{ lb}$ $\qquad b = 2\text{ ft}$

$L = 12\text{ ft}$ $\qquad \theta = 15\text{ deg}$

$a = 1\text{ ft}$



Solution:

Guess $\quad k = 1\,\frac{\text{lb}}{\text{ft}} \qquad d = 1\text{ ft}$

Given

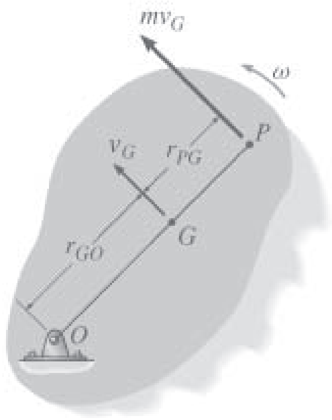
$$b^2 = \left(\frac{L}{2}\right)^2 + d^2 - 2d\left(\frac{L}{2}\right)\cos(\theta)$$

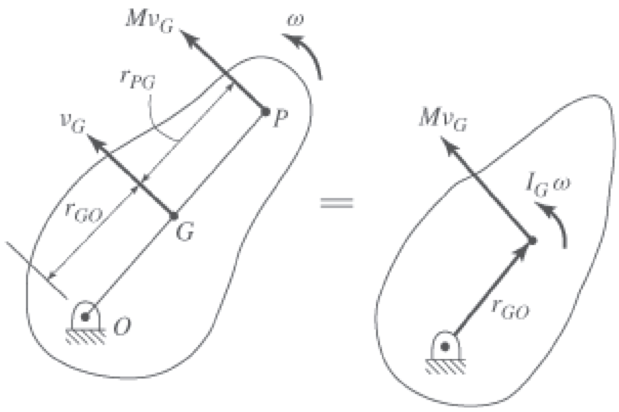
$$0 = -W\left(\frac{L}{2}\right) + 2\frac{1}{2}k\left(\frac{L}{2} + b - d\right)^2$$

$$\begin{pmatrix} k \\ d \end{pmatrix} = \text{Find}(k, d) \qquad d = 4.535\text{ ft} \qquad k = 100.0\,\frac{\text{lb}}{\text{ft}}$$

Problem 19-1

The rigid body (slab) has a mass m and is rotating with an angular velocity ω about an axis passing through the fixed point O. Show that the momenta of all the particles composing the body can be represented by a single vector having a magnitude mv_G and acting through point P, called the center of *percussion*, which lies at a distance $r_{PG} = k^2{}_G / r_{GO}$ from the mass center G. Here k_G is the radius of gyration of the body, computed about an axis perpendicular to the plane of motion and passing through G.





Solution:

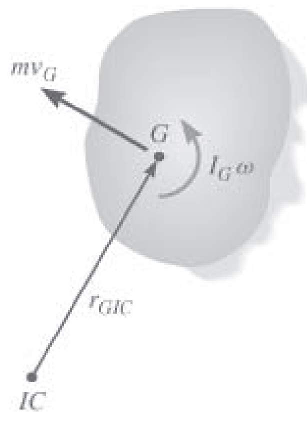
$$H_o = (r_{GO} + r_{PG}) m v_G = r_{GO} m v_G + I_G \omega \qquad \text{Where} \qquad I_G = m k_G^2$$

$$r_{GO} m v_G + r_{PG} m v_G = r_{GO} m v_G + m k_G^2 \omega$$

$$r_{PG} = \frac{k_G^2 \omega}{v_G} = \frac{k_G^2}{v_G}\left(\frac{v_G}{r_{GO}}\right) = \frac{k_G^2}{r_{GO}} \qquad \text{Q.E.D}$$

Problem 19-2

At a given instant, the body has a linear momentum $L = mv_G$ and an angular momentum $H_G = I_G\omega$ computed about its mass center. Show that the angular momentum of the body computed about the instantaneous center of zero velocity IC can be expressed as $H_{IC} = I_{IC}\omega$ where I_{IC} represents the body's moment of inertia computed about the instantaneous axis of zero velocity. As shown, the IC is located at a distance r_{GIC} away from the mass center G.



Solution:

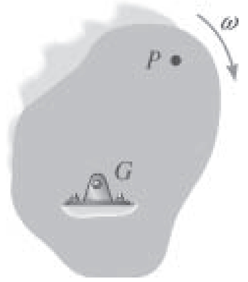
$$H_{IC} = r_{GIC} m v_G + I_G \omega \qquad \text{Where} \qquad v_G = \omega r_{GIC}$$

$$H_{IC} = r_{GIC} m \omega r_{GIC} + I_G \omega$$

$$H_{IC} = \left(I_G + m r_{GIC}^2\right)\omega$$

$$H_{IC} = I_{IC}\omega \qquad \text{Q.E.D.}$$

Problem 19-3



Show that if a slab is rotating about a fixed axis perpendicular to the slab and passing through its mass center *G,* the angular momentum is the same when computed about any other point *P* on the slab.

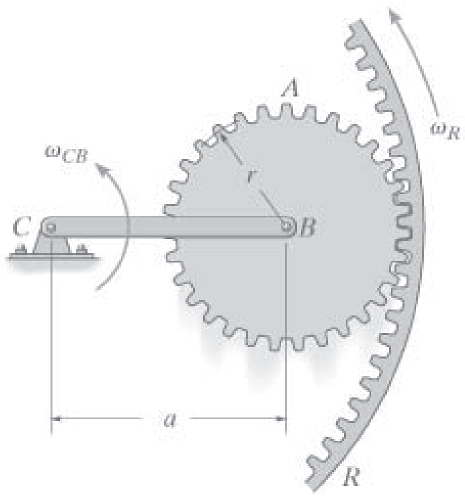
Solution:

Since $v_G = 0$, the linear momentum $L = mv_G = 0$. Hence the angular momentum about any point *P* is

$$H_P = I_G\omega$$

Since ω is a free vector , so is H_P. Q.E.D.

***Problem 19-4**



Gear *A* rotates along the inside of the circular gear rack *R*. If *A* has weight *W* and radius of gyration k_B, determine its angular momentum about point *C* when (a) $\omega_R = 0$, (b) $\omega_R = \omega$.

Given:

$W = 4\ \text{lbf}$ $\qquad r = 0.75\ \text{ft}$

$\omega_{CB} = 30\ \dfrac{\text{rad}}{\text{s}}$ $\qquad a = 1.5\ \text{ft}$

$\omega = 20\ \dfrac{\text{rad}}{\text{s}}$ $\qquad k_B = 0.5\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

Solution:

(a) $\omega_R = 0\ \dfrac{\text{rad}}{\text{s}}$

$$v_B = a\omega_{CB} \qquad \omega_A = \frac{\omega_R(a+r) - \omega_{CB}\,a}{r}$$

$$H_c = \left(\frac{W}{g}\right)v_B\,a + \left(\frac{W}{g}\right)k_B^2\,\omega_A \qquad H_c = 6.52\ \text{slug}\cdot\frac{\text{ft}^2}{\text{s}}$$

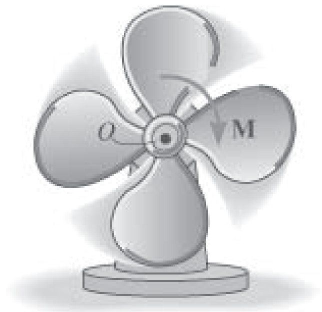
(b) $\omega_R = \omega$

$v_B = a\omega_{CB}$ $\qquad$ $\omega_A = \dfrac{\omega_R(a+r) - \omega_{CB}\,a}{r}$

$H_c = \left(\dfrac{W}{g}\right) v_B\,a + \left(\dfrac{W}{g}\right) k_B^{\,2}\,\omega_A$ $\qquad$ $H_c = 8.39\ \text{slug}\cdot\dfrac{\text{ft}^2}{\text{s}}$

Problem 19-5

The fan blade has mass m_b and a moment of inertia I_0 about an axis passing through its center O. If it is subjected to moment $M = A(1 - e^{bt})$ determine its angular velocity when $t = t_1$ starting from rest.



Given:

$m_b = 2\ \text{kg}$ $\qquad$ $A = 3\ \text{N}\cdot\text{m}$ $\qquad$ $t_1 = 4\ \text{s}$

$I_O = 0.18\ \text{kg}\cdot\text{m}^2$ $\qquad$ $b = -0.2\ \text{s}^{-1}$

Solution:

$0 + \displaystyle\int_0^{t_1} A\left(1 - e^{b\,t}\right)\,dt = I_O\omega_1$ $\qquad$ $\omega_1 = \dfrac{1}{I_O}\displaystyle\int_0^{t_1} A\left(1 - e^{b\,t}\right)\,dt$ $\qquad$ $\omega_1 = 20.8\ \dfrac{\text{rad}}{\text{s}}$

Problem 19-6

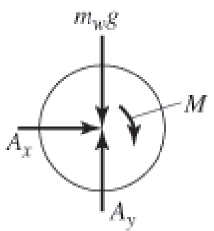
The wheel of mass m_w has a radius of gyration k_A. If the wheel is subjected to a moment $M = bt$, determine its angular velocity at time t_1 starting from rest. Also, compute the reactions which the fixed pin A exerts on the wheel during the motion.

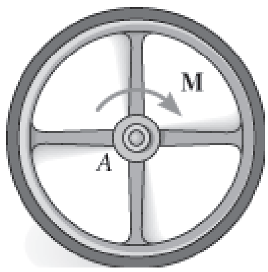
Given:

$m_w = 10\ \text{kg}$ $\qquad$ $t_1 = 3\ \text{s}$

$k_A = 200\ \text{mm}$ $\qquad$ $g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

$b = 5\ \text{N}\cdot\dfrac{\text{m}}{\text{s}}$





Solution:

$0 + \displaystyle\int_0^{t_1} b\,t\,dt = m_w k_A^{\,2}\omega_1$ $\qquad$ $\omega_1 = \dfrac{1}{m_w k_A^{\,2}}\displaystyle\int_0^{t_1} b\,t\,dt$ $\qquad$ $\omega_1 = 56.25\ \dfrac{\text{rad}}{\text{s}}$

$0 + Ax\,t_1 = 0$ $\qquad$ $A_x = 0$ $\qquad$ $A_x = 0.00$

$0 + A_y t_1 - m_W g t_1 = 0 \qquad A_y = m_W g \qquad A_y = 98.10\ \text{N}$

Problem 19-7

Disk D of weight W is subjected to counterclockwise moment $M = bt$. Determine the angular velocity of the disk at time t_2 after the moment is applied. Due to the spring the plate P exerts constant force P on the disk. The coefficients of static and kinetic friction between the disk and the plate are μ_s and μ_k respectively. *Hint*: First find the time needed to start the disk rotating.

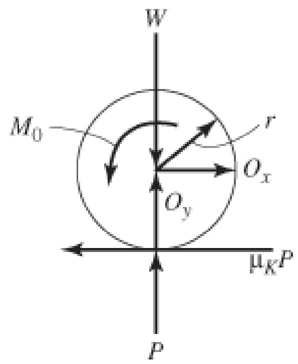
Given:

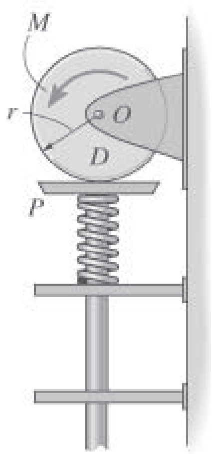
$W = 10\ \text{lb} \qquad \mu_s = 0.3$

$b = 10\ \text{lb}\cdot\frac{\text{ft}}{\text{s}} \qquad \mu_k = 0.2$

$t_2 = 2\ \text{s} \qquad r = 0.5\ \text{ft}$

$P = 100\text{lb} \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$





Solution: When motion begins

$$b t_1 = \mu_s P r \qquad t_1 = \frac{\mu_s P r}{b} \qquad t_1 = 1.50\ \text{s}$$

At a later time we have

$$0 + \int_{t_1}^{t_2} (b t - \mu_k P r)\, dt = \left(\frac{W}{g}\right)\frac{r^2}{2}\omega_2$$

$$\omega_2 = \frac{2g}{W r^2}\int_{t_1}^{t_2} (b t - \mu_k P r)\, dt \qquad \omega_2 = 96.6\ \frac{\text{rad}}{\text{s}}$$

***Problem 19-8**

The cord is wrapped around the inner core of the spool. If block B of weight W_B is suspended from the cord and released from rest, determine the spool's angular velocity when $t = t_1$. Neglect the mass of the cord. The spool has weight W_S and the radius of gyration about the axle A is k_A. Solve the problem in two ways, first by considering the "system" consisting of the block and spool, and then by considering the block and spool separately.

Given:

$W_B = 5\ \text{lb}$

$t_1 = 3\ \text{s}$

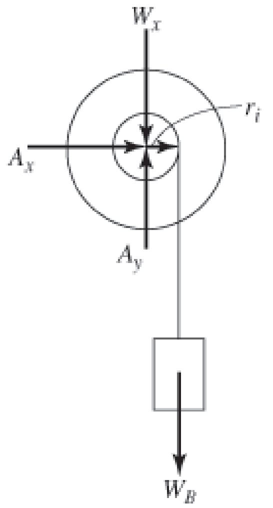
$W_S = 180\ \text{lb}$

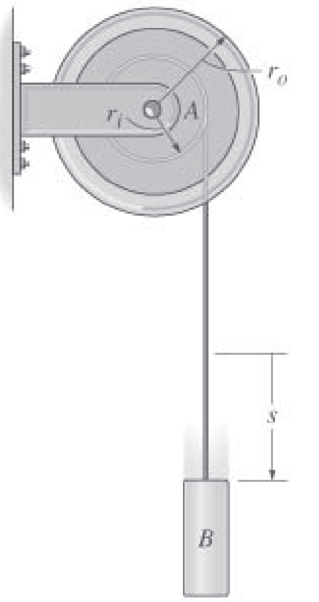
$k_A = 1.25\ \text{ft}$

$r_i = 1.5\text{ft}$

$r_o = 2.75\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$





Solution:

(a) System as a whole

$$0 + W_B r_i t_1 = \left(\frac{W_S}{g}\right) k_A^{\,2}\,\omega + \left(\frac{W_B}{g}\right)(r_i \omega) r_i \qquad \omega = \frac{W_B r_i t_1 g}{W_S k_A^{\,2} + W_B r_i^{\,2}} \qquad \omega = 2.48\ \frac{\text{rad}}{\text{s}}$$

(b) Parts separately Guesses $T = 1\ \text{lb}$ $\omega = 1\ \dfrac{\text{rad}}{\text{s}}$

Given $$0 + T t_1 r_i = \left(\frac{W_S}{g}\right) k_A^{\,2}\,\omega \qquad T t_1 - W_B t_1 = \left(\frac{-W_B}{g}\right)(r_i \omega)$$

$$\begin{pmatrix} T \\ \omega \end{pmatrix} = \text{Find}(T, \omega) \qquad T = 4.81\ \text{lb} \qquad \omega = 2.48\ \frac{\text{rad}}{\text{s}}$$

Problem 19-9

The disk has mass M and is originally spinning at the end of the strut with angular velocity ω. If it is then placed against the wall, for which the coefficient of kinetic friction is μ_k determine the time required for the motion to stop. What is the force in strut BC during this time?

Given:

$M = 20\ \text{kg}$ $\theta = 60\ \text{deg}$

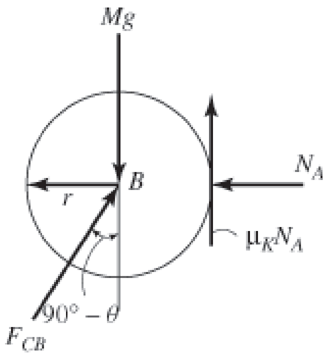
$\omega = 60\ \dfrac{\text{rad}}{\text{s}}$ $r = 150\ \text{mm}$

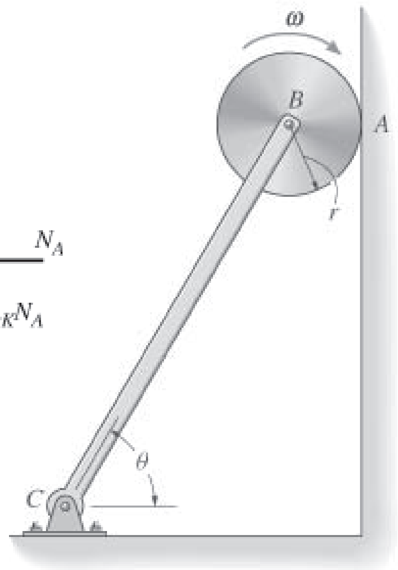
$\mu_k = 0.3$ $g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Solution:

Initial Guess:

$F_{CB} = 1\ \text{N}$ $t = 1\ \text{s}$ $N_A = 1\ \text{N}$





Given

$$-M\left(\frac{r^2}{2}\right)\omega + \mu_k N_A r t = 0$$

$$F_{CB}\cos(\theta)t - N_A t = 0$$

$$F_{CB}\sin(\theta)t - M g t + \mu_k N_A t = 0$$

$$\begin{pmatrix} F_{CB} \\ N_A \\ t \end{pmatrix} = \text{Find}(F_{CB}, N_A, t) \qquad N_A = 96.55\ \text{N} \qquad F_{CB} = 193\ \text{N} \qquad t = 3.11\ \text{s}$$

Problem 19-10

A flywheel has a mass M and radius of gyration k_G about an axis of rotation passing through its mass center. If a motor supplies a clockwise torque having a magnitude $M = kt$, determine the flywheel's angular veliocity at time t_1. Initially the flywheel is rotating clockwise at angular velocity ω_0.

Given:

$$M = 60\ \text{kg} \qquad k_G = 150\ \text{mm} \qquad k = 5\ \frac{\text{N}\cdot\text{m}}{\text{s}} \qquad t_1 = 3\ \text{s} \qquad \omega_0 = 2\ \frac{\text{rad}}{\text{s}}$$

Solution:

$$M k_G^2 \omega_0 + \int_0^{t_1} k t\ dt = M k_G^2 \omega_1$$

$$\omega_1 = \omega_0 + \frac{1}{M k_G^2}\int_0^{t_1} k t\ dt \qquad \omega_1 = 18.7\ \frac{\text{rad}}{\text{s}}$$

Problem 19-11

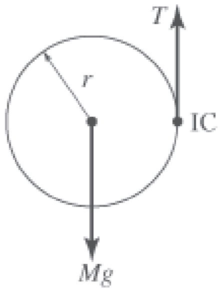
A wire of negligible mass is wrapped around the outer surface of the disk of mass M. If the disk is released from rest, determine its angular velocity at time t.

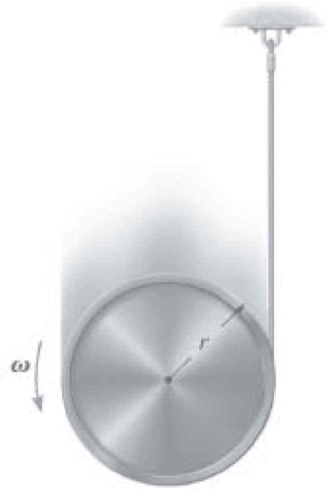
Given:

$M = 2\ \text{kg}$

$t = 3\ \text{s}$

$r = 80\ \text{mm}$





Solution:

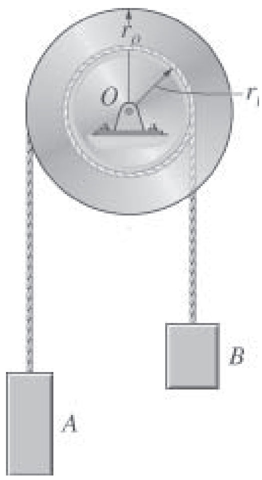
$$0 + Mgrt = \frac{3}{2}Mr^2\omega$$

$$\omega = \frac{2}{3}\left(\frac{g}{r}\right)t \qquad \omega = 245\ \frac{\text{rad}}{\text{s}}$$

***Problem 19-12**

The spool has mass m_S and radius of gyration k_O. Block A has mass m_A, and block B has mass m_B. If they are released from rest, determine the time required for block A to attain speed v_A. Neglect the mass of the ropes.

Given:



$m_S = 30\ \text{kg}$ $\quad m_B = 10\ \text{kg}$ $\quad r_o = 0.3\ \text{m}$

$k_O = 0.25\ \text{m}$ $\quad v_A = 2\ \frac{\text{m}}{\text{s}}$ $\quad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$m_A = 25\ \text{kg}$ $\quad r_i = 0.18\ \text{m}$

Solution:

Guesses $\quad t = 1\ \text{s} \quad v_B = 1\ \frac{\text{m}}{\text{s}} \quad \omega = 1\ \frac{\text{rad}}{\text{s}}$

Given $\quad v_A = \omega r_o \quad v_B = \omega r_i$

$$0 + m_A g t r_o - m_B g t r_i = m_A v_A r_o + m_B v_B r_i + m_S k_O^2 \omega$$

$$\begin{pmatrix} t \\ v_B \\ \omega \end{pmatrix} = \text{Find}(t, v_B, \omega) \qquad v_B = 1.20\ \frac{\text{m}}{\text{s}} \qquad \omega = 6.67\ \frac{\text{rad}}{\text{s}} \qquad t = 0.530\ \text{s}$$

Problem 19-13

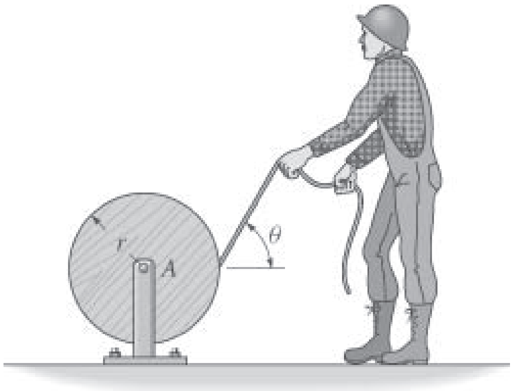
The man pulls the rope off the reel with a constant force P in the direction shown. If the reel has weight W and radius of gyration k_G about the trunnion (pin) at A, determine the angular velocity of the reel at time t starting from rest. Neglect friction and the weight of rope that is removed.

Given:

$P = 8\ \text{lb}$ $\quad t = 3\ \text{s}$ $\quad g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

$W = 250\ \text{lb}$ $\quad \theta = 60\ \text{deg}$

$k_G = 0.8\ \text{ft}$ $\quad r = 1.25\ \text{ft}$



Solution:

$$0 + P r t = \left(\frac{W}{g}\right) k_G^{\,2}\,\omega$$

$$\omega = \frac{P r t g}{W k_G^{\,2}} \qquad \omega = 6.04\ \frac{\text{rad}}{\text{s}}$$

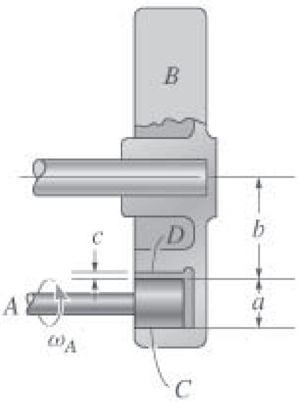
Problem 19-14

Angular motion is transmitted from a driver wheel A to the driven wheel B by friction between the wheels at C. If A always rotates at constant rate ω_A and the coefficient of kinetic friction between the wheels is μ_k, determine the time required for B to reach a constant angular velocity once the wheels make contact with a normal force F_N. What is the final angular velocity of wheel B? Wheel B has mass m_B and radius of gyration about its axis of rotation k_G.

Given:

$\omega_A = 16\ \dfrac{\text{rad}}{\text{s}}$ $\quad m_B = 90\ \text{kg}$ $\quad a = 40\ \text{mm}$ $\quad c = 4\ \text{mm}$

$\mu_k = 0.2$ $\quad k_G = 120\ \text{mm}$ $\quad b = 50\ \text{mm}$ $\quad F_N = 50\ \text{N}$



Solution: Guesses $t = 1\ \text{s}$ $\quad \omega_B = 1\ \dfrac{\text{rad}}{\text{s}}$

Given $\quad \mu_k F_N (a + b) t = m_B k_G^{\,2}\,\omega_B$

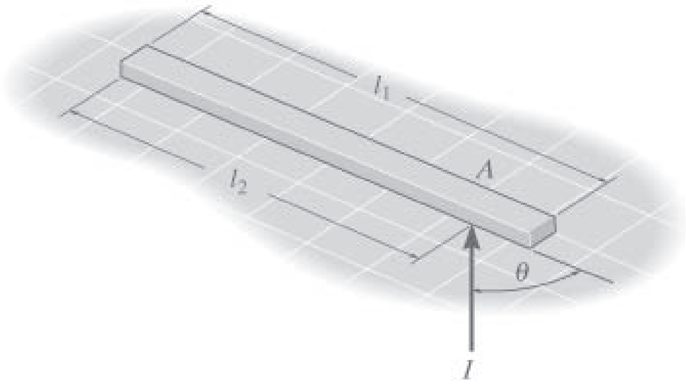
$$\omega_B (a + b) = \omega_A \left(\frac{a}{2}\right)$$

$$\begin{pmatrix} t \\ \omega_B \end{pmatrix} = \text{Find}(t, \omega_B) \qquad \omega_B = 3.56\ \frac{\text{rad}}{\text{s}} \qquad t = 5.12\ \text{s}$$

Problem 19-15

The slender rod of mass M rests on a smooth floor. If it is kicked so as to receive a horizontal impulse I at point A as shown, determine its angular velocity and the speed of its mass center.

Given:



$M = 4 \text{ kg}$

$l_1 = 2 \text{ m}$

$l_2 = 1.75 \text{ m}$

$I = 8 \text{ N s}$

$\theta = 60 \text{ deg}$

Solution:

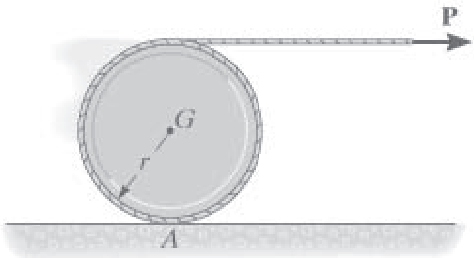
Guesses $v = 1 \frac{\text{m}}{\text{s}}$ $\omega = 1 \frac{\text{rad}}{\text{s}}$

Given $I \sin(\theta)\left(l_2 - \frac{l_1}{2}\right) = \frac{1}{12} M l_1^{\,2} \omega$ $I = M v$

$\begin{pmatrix} \omega \\ v \end{pmatrix} = \text{Find}(\omega, v)$ $\omega = 3.90 \frac{\text{rad}}{\text{s}}$ $v = 2.00 \frac{\text{m}}{\text{s}}$

***Problem 19-16**

A cord of negligible mass is wrapped around the outer surface of the cylinder of weight W and its end is subjected to a constant horizontal force **P**. If the cylinder rolls without slipping at A, determine its angular velocity in time t starting from rest. Neglect the thickness of the cord.



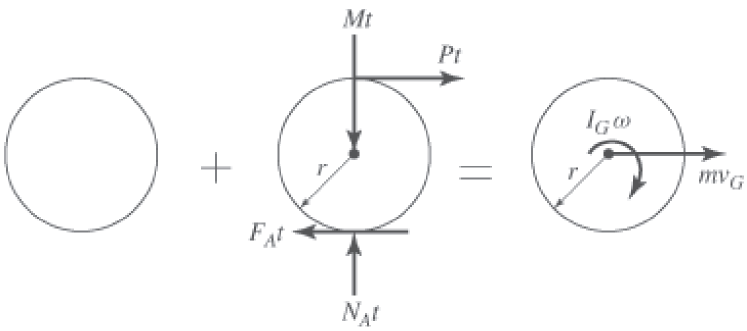
Given:

$W = 50 \text{ lb}$

$P = 2 \text{ lb}$

$t = 4 \text{ s}$

$r = 0.6 \text{ ft}$



Solution:

$$0 + P\,t(2r) = \left[\frac{1}{2}\left(\frac{W}{g}\right)r^2\right]\omega + \left(\frac{W}{g}\right)(r\omega)r$$

$$\omega = \frac{4P\,t\,g}{3r\,W} \qquad \omega = 11.4\,\frac{\text{rad}}{\text{s}}$$

Problem 19-17

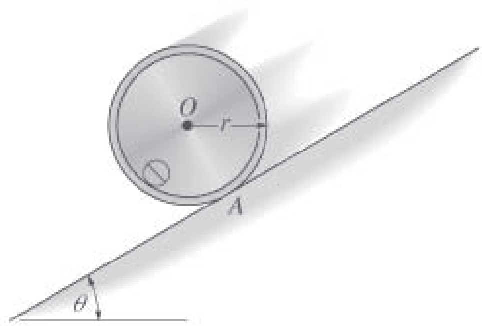
The drum has mass M, radius r, and radius of gyration k_O. If the coefficients of static and kinetic friction at A are μ_s and μ_k respectively, determine the drum's angular velocity at time t after it is released from rest.

Given:

$M = 70\text{ kg}$ $\quad \mu_s = 0.4$ $\quad \theta = 30\text{ deg}$

$r = 300\text{mm}$ $\quad \mu_k = 0.3$ $\quad g = 9.81\,\frac{\text{m}}{\text{s}^2}$

$k_O = 125\text{ mm}$ $\quad t = 2\text{ s}$



Solution: Assume no slip

Guesses $\quad F_f = 1\text{ N} \quad F_N = 1\text{ N}$

$\omega = 1\,\frac{\text{rad}}{\text{s}} \quad v = 1\,\frac{\text{m}}{\text{s}} \quad F_{max} = 1\text{ N}$

Given $\quad 0 + F_f r\,t = M k_O^2\,\omega \qquad v = \omega r \qquad F_{max} = \mu_s F_N$

$M g\sin(\theta)t - F_f t = M v \qquad F_N t - M g\cos(\theta)t = 0$

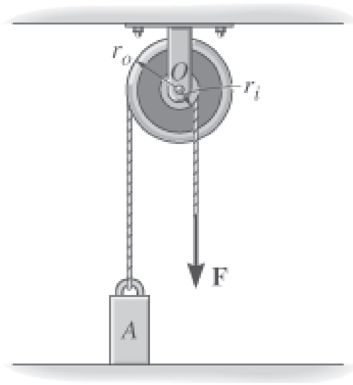
$$\begin{pmatrix} F_f \\ F_{max} \\ F_N \\ \omega \\ v \end{pmatrix} = \text{Find}(F_f, F_{max}, F_N, \omega, v) \qquad \begin{pmatrix} F_f \\ F_{max} \\ F_N \end{pmatrix} = \begin{pmatrix} 51 \\ 238 \\ 595 \end{pmatrix}\text{N} \qquad v = 8.36\,\frac{\text{m}}{\text{s}}$$

$$\omega = 27.9\,\frac{\text{rad}}{\text{s}}$$

Since $F_f = 51\text{ N} < F_{max} = 238\text{ N}$ then our no-slip assumption is good.

Problem 19-18

The double pulley consists of two wheels which are attached to one another and turn at the same rate. The pulley has mass m_p and radius of gyration k_O. If the block at A has mass m_A, determine the speed of the block at time t after a constant force **F** is applied to the rope wrapped around the inner hub of the pulley. The block is originally at rest. Neglect the mass of the rope.



Units Used: $\quad kN = 10^3\ N$

Given:

$m_p = 15\ kg \qquad F = 2\ kN$

$k_O = 110\ mm \qquad r_i = 75\ mm$

$m_A = 40\ kg \qquad r_o = 200\ mm$

$t = 3\ s$

Solution: Guess $\quad v_A = 1\ \frac{m}{s} \qquad \omega = 1\ \frac{rad}{s}$

Given $\quad -F r_i t + m_A g r_o t = -m_p k_O^2 \omega - m_A v_A r_o$

$v_A = \omega r_o$

$\begin{pmatrix} v_A \\ \omega \end{pmatrix} = \text{Find}(v_A, \omega) \qquad \omega = 120.44\ \frac{rad}{s} \qquad v_A = 24.1\ \frac{m}{s}$

Problem 19-19

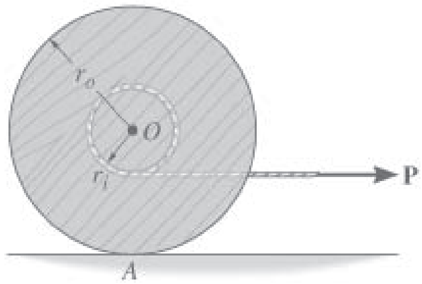
The spool has weight W and radius of gyration k_O. A cord is wrapped around its inner hub and the end subjected to a horizontal force **P**. Determine the spool's angular velocity at time t starting from rest. Assume the spool rolls without slipping.

Given:

$W = 30\ lb \qquad t = 4\ s$

$k_O = 0.45\ ft \qquad r_i = 0.3\ ft$

$P = 5\ lb \qquad r_o = 0.9\ ft$



Solution: Guesses $\quad \omega = 1\ \frac{rad}{s} \qquad v_O = 1\ \frac{ft}{s}$

Given $\quad -P(r_o - r_i)t = \left(\frac{-W}{g}\right)v_O r_o - \left(\frac{W}{g}\right)k_O^2 \omega \qquad v_O = \omega r_o$

$\begin{pmatrix} v_O \\ \omega \end{pmatrix} = \text{Find}(v_O, \omega) \qquad v_O = 3.49\ \frac{m}{s} \qquad \omega = 12.7\ \frac{rad}{s}$

***Problem 19-20**

The two gears A and B have weights W_A, W_B and radii of gyration k_A and k_B respectively. If a motor transmits a couple moment to gear B of $M = M_0(1 - e^{-bt})$, determine the angular velocity of gear A

at time t, starting from rest.

Given:

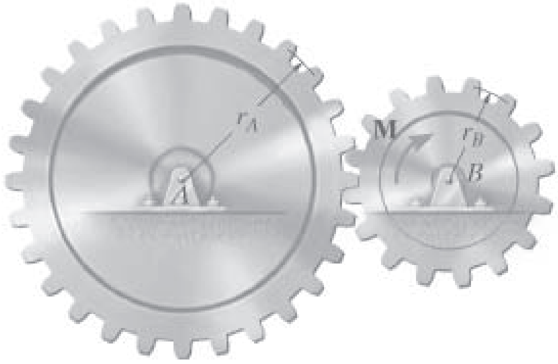
$W_A = 15\ \text{lb}$ $W_B = 10\ \text{lb}$

$r_A = 0.8\ \text{ft}$ $r_B = 0.5\ \text{ft}$

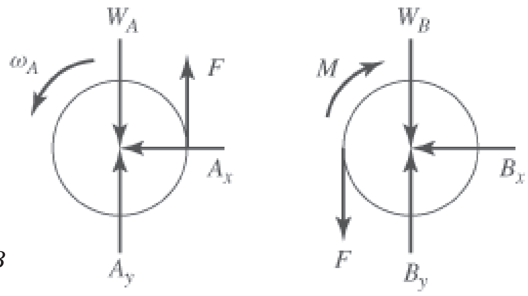
$k_A = 0.5\ \text{ft}$ $k_B = 0.35\ \text{ft}$

$M_0 = 2\ \text{lb}\cdot\text{ft}$ $b = 0.5\ \text{s}^{-1}$

$t = 5\ \text{s}$



Solution:



Guesses

$\omega_A = 1\ \dfrac{\text{rad}}{\text{s}}$ $\omega_B = 1\ \dfrac{\text{rad}}{\text{s}}$ $ImpF = 1\ \text{lb}\cdot\text{s}$

Given $\displaystyle\int_0^t M_0\left(1 - e^{-b\,t}\right) dt - ImpF\,r_B = \left(\frac{W_B}{g}\right) k_B^{\,2}\,\omega_B$

$ImpF\,r_A = \left(\dfrac{W_A}{g}\right) k_A^{\,2}\,\omega_A$ $\omega_A\,r_A = \omega_B\,r_B$

$\begin{pmatrix} \omega_A \\ \omega_B \\ ImpF \end{pmatrix} = \text{Find}\left(\omega_A, \omega_B, ImpF\right)$ $ImpF = 6.89\ \text{lb s}$ $\omega_B = 75.7\ \dfrac{\text{rad}}{\text{s}}$

$\omega_A = 47.3\ \dfrac{\text{rad}}{\text{s}}$

Problem 19-21

Spool B is at rest and spool A is rotating at ω when the slack in the cord connecting them is taken up. Determine the angular velocity of each spool immediately after the cord is jerked tight by the spinning of spool A. The weights and radii of gyration of A and B are W_A, k_A, and W_B, k_A, respectively.

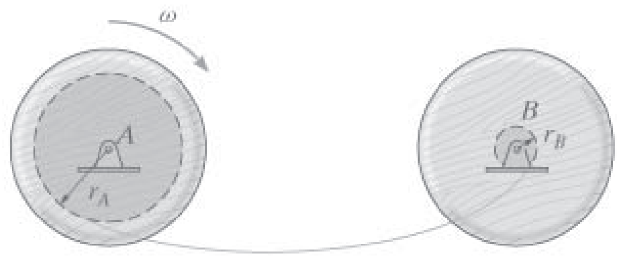
Given:

$W_A = 30\ \text{lb}$ $\quad W_B = 15\ \text{lb}$

$k_A = 0.8\ \text{ft}$ $\quad k_B = 0.6\ \text{ft}$

$r_A = 1.2\ \text{ft}$ $\quad r_B = 0.4\ \text{ft}$

$\omega = 6\ \dfrac{\text{rad}}{\text{s}}$

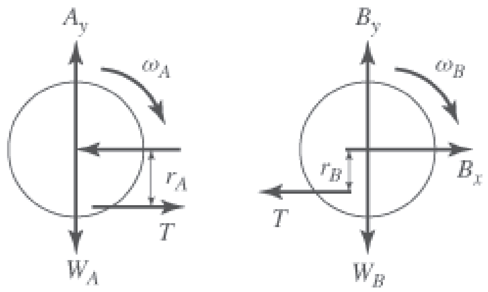


Solution:

Guesses

$\omega_A = 1\ \dfrac{\text{rad}}{\text{s}}$ $\quad \omega_B = 1\ \dfrac{\text{rad}}{\text{s}}$

$Imp = 1\ \text{lb}\cdot\text{s}$



Given

$$\left(\frac{W_A}{g}\right)k_A^2\,\omega - Imp\, r_A = \left(\frac{W_A}{g}\right)k_A^2\,\omega_A \qquad Imp\, r_B = \left(\frac{W_B}{g}\right)k_B^2\,\omega_B \qquad \omega_A r_A = \omega_B r_B$$

$$\begin{pmatrix}\omega_A\\ \omega_B\\ Imp\end{pmatrix} = \text{Find}(\omega_A, \omega_B, Imp) \qquad Imp = 2.14\ \text{lb}\cdot\text{s} \qquad \begin{pmatrix}\omega_A\\ \omega_B\end{pmatrix} = \begin{pmatrix}1.70\\ 5.10\end{pmatrix}\frac{\text{rad}}{\text{s}}$$

Problem 19-22

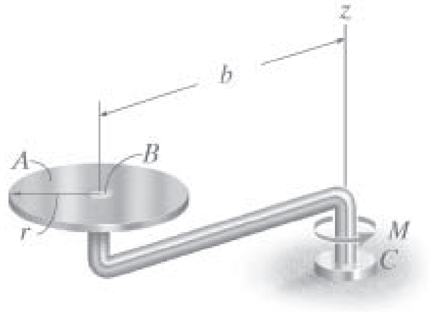
Disk *A* of mass m_A is mounted on arm *BC*, which has a negligible mass. If a torque of $M = M_0e^{at}$ is applied to the arm at *C*, determine the angular velocity of *BC* at time *t* starting from rest. Solve the problem assuming that (a) the disk is set in a smooth bearing at *B* so that it rotates with curvilinear translation, (b) the disk is fixed to the shaft *BC*, and (c) the disk is given an initial freely spinning angular velocity $\omega_D\mathbf{k}$ prior to application of the torque.

Given:

$m_A = 4 \text{ kg}$ $M_0 = 5 \text{ N·m}$ $\omega_D = -80 \frac{\text{rad}}{\text{s}}$

$r = 60 \text{ mm}$ $a = 0.5 \text{ s}^{-1}$

$b = 250 \text{ mm}$ $t = 2 \text{ s}$



Solution: Guess $\omega_{BC} = 1 \frac{\text{rad}}{\text{s}}$

(a) Given $\int_0^t M_0 e^{a t}\, dt = m_A \omega_{BC} b^2$

$\omega_{BC} = \text{Find}(\omega_{BC})$ $\omega_{BC} = 68.7 \frac{\text{rad}}{\text{s}}$

(b) Given $\int_0^t M_0 e^{a t}\, dt = m_A \omega_{BC} b^2 + m_A \left(\frac{r^2}{2}\right) \omega_{BC}$

$\omega_{BC} = \text{Find}(\omega_{BC})$ $\omega_{BC} = 66.8 \frac{\text{rad}}{\text{s}}$

(c) Given $-m_A \left(\frac{r^2}{2}\right) \omega_D + \int_0^t M_0 e^{a t}\, dt = m_A \omega_{BC} b^2 - m_A \left(\frac{r^2}{2}\right) \omega_D$

$\omega_{BC} = \text{Find}(\omega_{BC})$ $\omega_{BC} = 68.7 \frac{\text{rad}}{\text{s}}$

Problem 19-23

The inner hub of the wheel rests on the horizontal track. If it does not slip at A, determine the speed of the block of weight W_b at time t after the block is released from rest. The wheel has weight W_w and radius of gyration k_G. Neglect the mass of the pulley and cord.

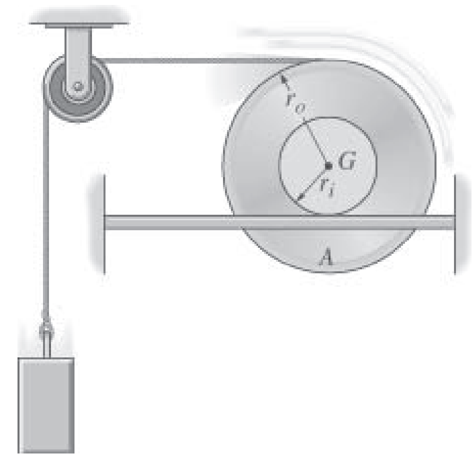
Given:

$W_b = 10 \text{ lb}$ $r_i = 1 \text{ ft}$

$t = 2 \text{ s}$ $r_o = 2 \text{ ft}$

$W_w = 30 \text{ lb}$ $g = 32.2 \frac{\text{ft}}{\text{s}^2}$

$k_G = 1.30 \text{ ft}$



Solution: Guesses $v_G = 1 \frac{\text{ft}}{\text{s}}$

$v_B = 1 \frac{\text{ft}}{\text{s}}$ $T = 1 \text{ lb}$ $\omega = 1 \frac{\text{rad}}{\text{s}}$

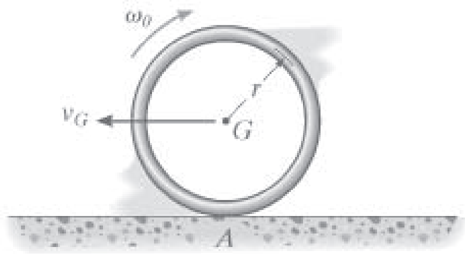
Given $\quad T\,t - W_b\,t = \left(\frac{-W_b}{g}\right)v_B \qquad T(r_o + r_i)t = \left(\frac{W_w}{g}\right)v_G r_i + \left(\frac{W_w}{g}\right)k_G^{\,2}\,\omega$

$$v_G = \omega r_i \qquad v_B = \omega(r_i + r_o)$$

$$\begin{pmatrix} v_G \\ v_B \\ \omega \\ T \end{pmatrix} = \text{Find}(v_G, v_B, \omega, T) \qquad T = 4.73\text{ lb} \quad \omega = 11.3\,\frac{\text{rad}}{\text{s}} \qquad v_G = 11.3\,\frac{\text{ft}}{\text{s}} \qquad v_B = 34.0\,\frac{\text{ft}}{\text{s}}$$

***Problem 19-24**

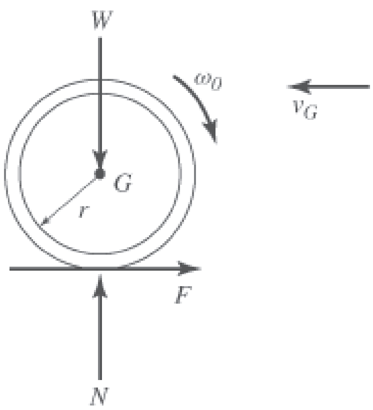
If the hoop has a weight W and radius r and is thrown onto a *rough surface* with a velocity v_G parallel to the surface, determine the amount of backspin, ω_0, it must be given so that it stops spinning at the same instant that its forward velocity is zero. It is not necessary to know the coefficient of kinetic friction at A for the calculation.



Solution:

$$\left(\frac{W}{g}\right)v_G r - \left(\frac{W}{g}\right)r^2\,\omega_0 = 0$$

$$\omega_0 = \frac{v_G}{r}$$



Problem 19-25

The rectangular plate of weight W is at rest on a smooth *horizontal* floor. If it is given the horizontal impulses shown, determine its angular velocity and the velocity of the mass center.

Given:

$W = 10 \text{ lb}$ $\quad d = 0.5 \text{ ft}$

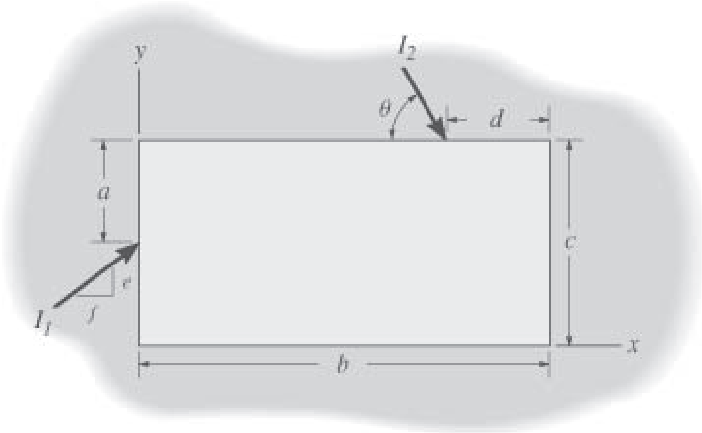
$I_1 = 20 \text{ lb}\cdot\text{s}$ $\quad \theta = 60 \text{ deg}$

$I_2 = 5 \text{ lb}\cdot\text{s}$ $\quad e = 3$

$a = 0.5 \text{ ft}$ $\quad f = 4$

$b = 2 \text{ ft}$ $\quad g = 32.2 \frac{\text{ft}}{\text{s}^2}$

$c = 1 \text{ ft}$



Solution: Guesses $v_x = 1 \frac{\text{ft}}{\text{s}}$ $\quad v_y = 1 \frac{\text{ft}}{\text{s}}$ $\quad \omega = 1 \frac{\text{rad}}{\text{s}}$

Given

$$\left(\frac{f}{\sqrt{e^2 + f^2}}\right) I_1 + I_2 \cos(\theta) = \left(\frac{W}{g}\right) v_x$$

$$\left(\frac{e}{\sqrt{e^2 + f^2}}\right) I_1 - I_2 \sin(\theta) = \left(\frac{W}{g}\right) v_y$$

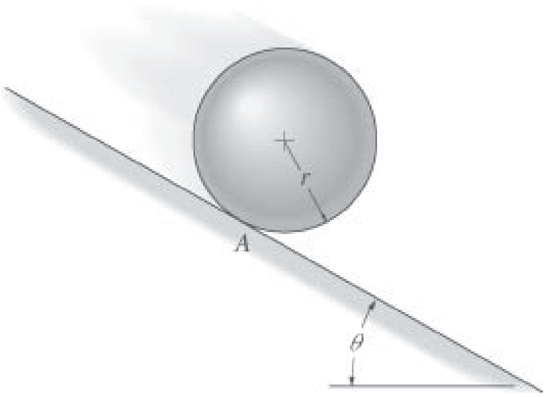
$$\left(\frac{f}{\sqrt{e^2 + f^2}}\right) I_1 a - I_2 \sin(\theta)(b - d) = \frac{1}{12}\left(\frac{W}{g}\right)\left(b^2 + c^2\right)\omega + \left(\frac{W}{g}\right) v_x \left(\frac{c}{2}\right) + \left(\frac{W}{g}\right) v_y \left(\frac{b}{2}\right)$$

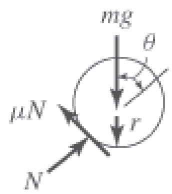
$$\begin{pmatrix} v_x \\ v_y \\ \omega \end{pmatrix} = \text{Find}(v_x, v_y, \omega) \qquad \mathbf{v_G} = \begin{pmatrix} v_x \\ v_y \end{pmatrix} \qquad \omega = -119 \frac{\text{rad}}{\text{s}} \qquad \mathbf{v_G} = \begin{pmatrix} 59.6 \\ 24.7 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

$$\left|\mathbf{v_G}\right| = 64.5 \frac{\text{ft}}{\text{s}}$$

Problem 19-26

The ball of mass m and radius r rolls along an inclined plane for which the coefficient of static friction is μ. If the ball is released from rest, determine the maximum angle θ for the incline so that it rolls without slipping at A.





Solution:

$$N t - m g \cos(\theta) t = 0 \qquad N = m g \cos(\theta)$$

$$\mu N r t = \frac{2}{5} m r^2 \omega \qquad t = \frac{2 \omega r}{5 \mu g \cos(\theta)}$$

$$m g \sin(\theta) t - \mu N t = m r \omega \qquad t = \frac{r \omega}{g(\sin(\theta) - \mu \cos(\theta))}$$

Thus

$$\frac{2 \omega r}{5 \mu g \cos(\theta)} = \frac{r \omega}{g(\sin(\theta) - \mu \cos(\theta))} \qquad \theta = \operatorname{atan}\left(\frac{7\mu}{2}\right)$$

Problem 19-27

The spool has weight W_s and radius of gyration k_O. If the block B has weight W_b and a force **P** is applied to the cord, determine the speed of the block at time t starting from rest. Neglect the mass of the cord.

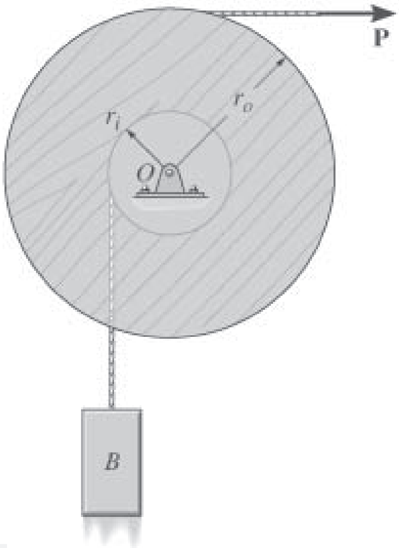
Given:

$W_s = 75 \text{lb} \qquad t = 5 \text{s}$

$k_O = 1.2\text{ft}$ $\quad r_o = 2\text{ft}$

$W_b = 60\text{lb}$ $\quad r_i = 0.75\text{ft}$

$P = 25\text{lb}$



Solution:

Guess $\quad v_B = 1\dfrac{\text{ft}}{\text{s}} \qquad \omega = 1\dfrac{\text{rad}}{\text{s}}$

Given

$$-P\,r_o\,t + W_b\,r_i\,t = \left(\frac{-W_b}{g}\right)v_B\,r_i - \left(\frac{W_s}{g}\right)k_O^{\,2}\,\omega$$

$$v_B = \omega r_i$$

$$\begin{pmatrix} v_B \\ \omega \end{pmatrix} = \text{Find}(v_B, \omega) \qquad \omega = 5.68\,\frac{\text{rad}}{\text{s}} \qquad v_B = 4.26\,\frac{\text{ft}}{\text{s}}$$

***Problem 19-28**

The slender rod has a mass m and is suspended at its end A by a cord. If the rod receives a horizontal blow giving it an impulse **I** at its bottom B, determine the location y of the point P about which the rod appears to rotate during the impact.

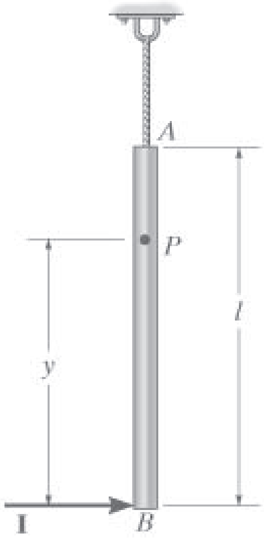
Solution:

$$F\Delta t = m\,v_{cm}$$

$$m\Delta t = I\omega$$

$$I\,y = I'\omega$$

$$I\,y = m\left[\frac{l^2}{12} + \left(y - \frac{l}{2}\right)^2\right]\omega \qquad (1)$$



$$I = m\,v_{cm}$$

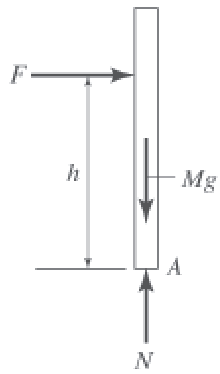
$$I = m\frac{l}{2}\omega \qquad (2)$$

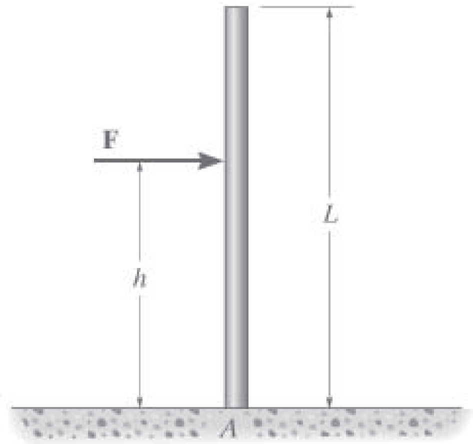
Divide Eqs (1) by (2)

$$\frac{y\,l}{2} = \left[\frac{l^2}{12} + \left(y - \frac{l}{2}\right)^2\right] \qquad y = \left(\frac{3}{4} + \frac{\sqrt{33}}{12}\right)l$$

Problem 19-29

A thin rod having mass M is balanced vertically as shown. Determine the height h at which it can be struck with a horizontal force **F** and not slip on the floor. This requires that the frictional force at A be essentially zero.





Given:

$M = 4\ \text{kg} \qquad L = 0.8\ \text{m}$

Solution:

$$F\,t = M\omega\frac{L}{2}$$

$$F\,t\,h = M\frac{L^2}{3}\omega$$

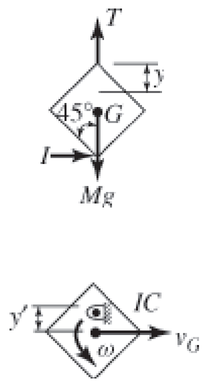
Thus

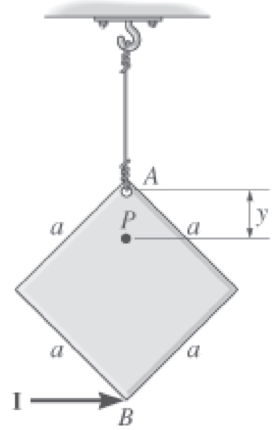
$$M\omega\frac{L}{2}h = M\frac{L^2}{3}\omega$$

$$h = \frac{2}{3}L \qquad h = 0.53\ \text{m}$$

Problem 19-30

The square plate has a mass M and is suspended at its corner A by a cord. If it receives a horizontal impulse **I** at corner B, determine the location y' of the point P about which the plate appears to rotate during the impact.





Solution:

$$I\frac{a}{\sqrt{2}} = \frac{1}{6}Ma^2\omega \qquad \omega = \frac{6}{\sqrt{2}}\frac{I}{Ma}$$

$$I = Mv_G \qquad v_G = \frac{I}{M}$$

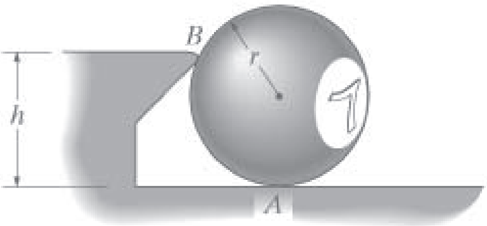
$$y' = \frac{v_G}{\omega} \qquad y' = \frac{\sqrt{2}}{6}a$$

$$y = \frac{a}{\sqrt{2}} - y' \qquad y = \frac{\sqrt{2}}{3}a$$

Problem 19-31

Determine the height h of the bumper of the pool table, so that when the pool ball of mass m strikes it, no frictional force will be developed between the ball and the table at A. Assume the bumper exerts only a horizontal force on the ball.

Solution:

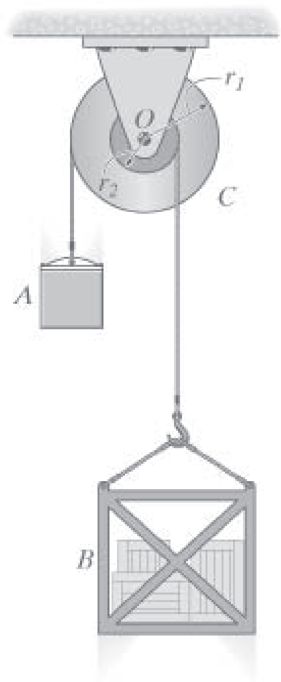


$$F\Delta t = M\Delta v \qquad F\Delta t\, h = \frac{7}{5} M r^2 \Delta\omega \qquad \Delta v = r\Delta\omega$$

Thus

$$M r \Delta\omega h = \frac{7}{5} M r^2 \Delta\omega \qquad h = \frac{7}{5} r$$

***Problem 19-32**



The double pulley consists of two wheels which are attached to one another and turn at the same rate. The pulley has a mass M_C and a radius of gyration k_O. If the block at A has a mass M_A and the container at B has a mass M_B, including its contents, determine the speed of the container at time t after it is released from rest.

Given:

$M_C = 15$ kg $\quad k_O = 110$ mm

$M_A = 40$ kg $\quad r_1 = 200$ mm

$M_B = 85$ kg $\quad r_2 = 75$ mm

$t = 3$ s

Solution:

Guess $\quad v_A = 1\frac{\text{m}}{\text{s}} \qquad v_B = 1\frac{\text{m}}{\text{s}} \qquad \omega = 1\frac{\text{rad}}{\text{s}}$

Given $\quad M_A g t r_1 - M_B g t r_2 = M_A v_A r_1 + M_B v_B r_2 + M_C k_O^2 \omega$

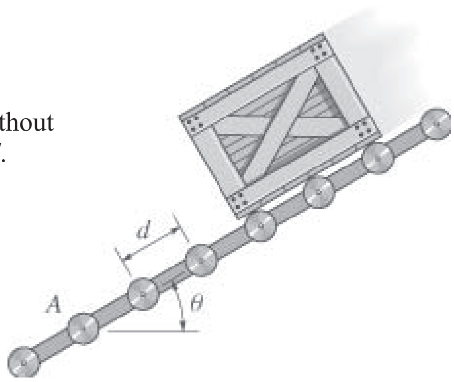
$v_A = \omega r_1 \qquad v_B = \omega r_2$

$$\begin{pmatrix} v_A \\ v_B \\ \omega \end{pmatrix} = \text{Find}(v_A, v_B, \omega) \qquad \omega = 21.2\frac{\text{rad}}{\text{s}} \qquad v_A = 4.23\frac{\text{m}}{\text{s}} \qquad v_B = 1.59\frac{\text{m}}{\text{s}}$$

Problem 19-33

The crate has a mass M_c. Determine the constant speed v_0 it acquires as it moves down the conveyor. The rollers each have radius r, mass M, and are spaced distance d apart. Note that friction causes each roller to rotate when the crate comes in contact with it.

Solution:



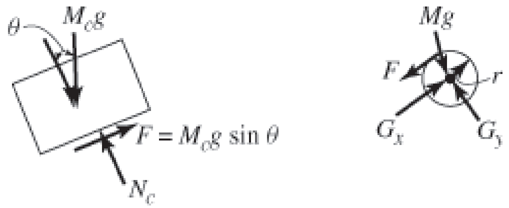
Assume each roller is brought to the condition of roll without slipping. In time t, the number of rollers affected is v_0t/d.

$$M_C g \sin(\theta) t - F t = 0$$

$$F = M_C g \sin(\theta)$$

$$F t r = \left(\frac{1}{2} M r^2\right) \frac{v_0}{r} \left(\frac{v_0}{d} t\right)$$

$$v_0 = \sqrt{2g \sin(\theta) d \frac{M_C}{M}}$$



Problem 19-34

Two wheels A and B have masses m_A and m_B and radii of gyration about their central vertical axes of k_A and k_B respectively. If they are freely rotating in the same direction at ω_A and ω_B about the same vertical axis, determine their common angular velocity after they are brought into contact and slipping between them stops.

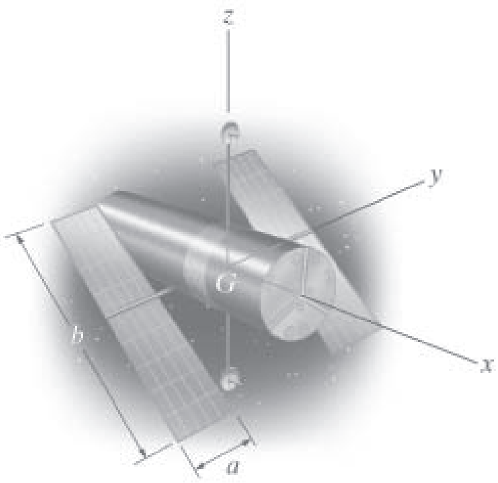
Solution:

$$m_A k_A^2 \omega_A + m_B k_B^2 \omega_B = \left(m_A k_A^2 + m_B k_B^2\right) \omega$$

$$\omega = \frac{m_A k_A^2 \omega_A + m_B k_B^2 \omega_B}{m_A k_A^2 + m_B k_B^2}$$

Problem 19-35

The Hubble Space Telescope is powered by two solar panels as shown. The body of the telescope has a mass M_1 and radii of gyration k_x and k_y, whereas the solar panels can be considered as thin plates, each having a mass M_2. Due to an internal drive, the panels are given an angular velocity of $\omega_0\mathbf{j}$, measured relative to the telescope. Determine the angular velocity of the telescope due to the rotation of the panels. Prior to rotating the panels, the telescope was originally traveling at $\mathbf{v_G} = (v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k})$. Neglect its orbital rotation.



Units Used: $\mathrm{Mg} = 10^3 \mathrm{kg}$

Given:

$M_1 = 11\ \mathrm{Mg}$ $\quad \omega_0 = 0.6\ \frac{\mathrm{rad}}{\mathrm{s}}$ $\quad v_x = -400\ \frac{\mathrm{m}}{\mathrm{s}}$

$M_2 = 54\ \mathrm{kg}$ $\quad a = 1.5\ \mathrm{m}$ $\quad v_y = 250\ \frac{\mathrm{m}}{\mathrm{s}}$

$k_x = 1.64\ \mathrm{m}$

$k_y = 3.85\ \mathrm{m}$ $\quad b = 6\ \mathrm{m}$ $\quad v_z = 175\ \frac{\mathrm{m}}{\mathrm{s}}$

Solution: Angular momentum is conserved.

Guess $\omega_T = 1\ \frac{\mathrm{rad}}{\mathrm{s}}$

Given $0 = 2\left(\frac{1}{12} M_2 b^2\right)(\omega_0 - \omega_T) - \left(M_1 k_y^2\right)\omega_T$ $\quad \omega_T = \mathrm{Find}(\omega_T)$

$\omega_T = 0.00\ \frac{\mathrm{rad}}{\mathrm{s}}$

***Problem 19-36**

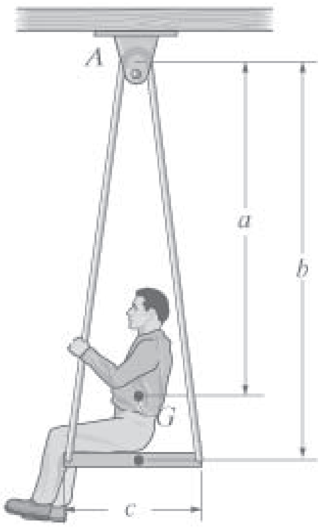
The platform swing consists of a flat plate of weight W_p suspended by four rods of negligible weight. When the swing is at rest, the man of weight W_m jumps off the platform when his center of gravity G is at distance a from the pin at A. This is done with a horizontal velocity v, measured relative to the swing at the level of G. Determine the angular velocity he imparts to the swing just after jumping off.

Given:

$W_p = 200\mathrm{lb}$ $\quad a = 10\mathrm{ft}$

$W_m = 150\mathrm{lb}$ $\quad b = 11\mathrm{ft}$

$v = 5\ \frac{\mathrm{ft}}{\mathrm{s}}$ $\quad c = 4\mathrm{ft}$



Solution:

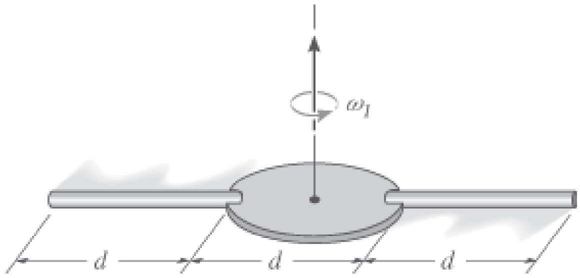
Guess $\omega = 1\ \frac{\mathrm{rad}}{\mathrm{s}}$

Given $0 = \frac{-W_m}{g}(v - \omega a)a + \frac{W_p}{g}\left(\frac{c^2}{12} + b^2\right)\omega$

$\omega = \mathrm{Find}(\omega)$ $\quad \omega = 0.190\ \frac{\mathrm{rad}}{\mathrm{s}}$

Problem 19-37

Each of the two slender rods and the disk have the same mass *m*. Also, the length of each rod is equal to the diameter *d* of the disk. If the assembly is rotating with an angular velocity ω_1 when the rods are directed outward, determine the angular velocity of the assembly if by internal means the rods are brought to an upright vertical position.



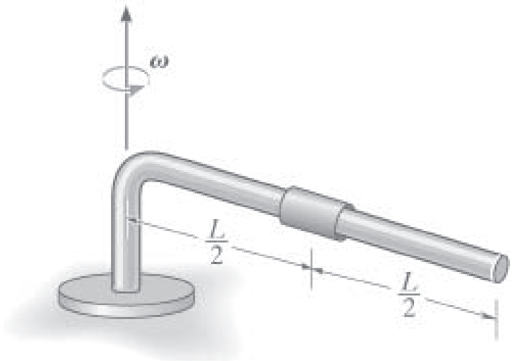
Solution:

$$H_1 = H_2$$

$$\left[\frac{1}{2}m\left(\frac{d}{2}\right)^2 + 2\frac{1}{12}m\,d^2 + 2m\,d^2\right]\omega_1 = \left[\frac{1}{2}m\left(\frac{d}{2}\right)^2 + 2m\left(\frac{d}{2}\right)^2\right]\omega_2 \qquad \omega_2 = \frac{11}{3}\omega_1$$

Problem 19-38

The rod has a length *L* and mass *m*. A smooth collar having a negligible size and one-fourth the mass of the rod is placed on the rod at its midpoint. If the rod is freely rotating with angular velocity ω about its end and the collar is released, determine the rod's angular velocity just before the collar flies off the rod. Also, what is the speed of the collar as it leaves the rod?



Solution:

$$H_1 = H_2$$

$$\frac{1}{3}m\,L^2\omega + \left(\frac{m}{4}\right)\left(\frac{L}{2}\right)\omega\left(\frac{L}{2}\right) = \frac{1}{3}m\,L^2\omega' + \left(\frac{m}{4}\right)L\,\omega'\,L \qquad \omega' = \frac{19}{28}\omega$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left(\frac{1}{3}m\,L^2\right)\omega^2 + \frac{1}{2}\left(\frac{m}{4}\right)\left(\frac{L}{2}\omega\right)^2 = \frac{1}{2}\left(\frac{m}{4}\right)v'^2 + \frac{1}{2}\left(\frac{m}{4}\right)(L\omega')^2 + \frac{1}{2}\left(\frac{1}{3}m\,L^2\right)\omega'^2$$

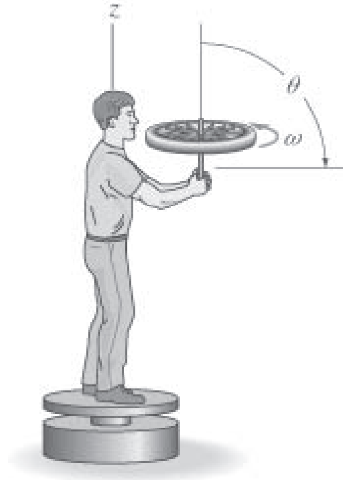
$$v'^2 = \frac{57}{112}L^2\,\omega^2$$

$$v'' = \sqrt{\frac{57}{112}L^2\,\omega^2 + \left[L\left(\frac{19}{28}\omega\right)\right]^2} \qquad v'' = \sqrt{\frac{95}{98}}\,\omega L \qquad v'' = 0.985\,\omega L$$

Problem 19-39

A man has a moment of inertia I_z about the z axis. He is originally at rest and standing on a small platform which can turn freely. If he is handed a wheel which is rotating at angular velocity ω and has a moment of inertia I about its spinning axis, determine his angular velocity if (a) he holds the wheel upright as shown, (b) turns the wheel out $\theta = 90°$, and (c) turns the wheel downward. $\theta = 180°$.

Solution:



(a) $0 + I\omega = I_z\omega_M + I\omega$ $\qquad \omega_M = 0$

(b) $0 + I\omega = I_z\omega_M + 0$ $\qquad \omega_M = \dfrac{I}{I_z}\omega$

(c) $0 + I\omega = I_z\omega_M - I\omega$ $\qquad \omega_M = \dfrac{2I}{I_z}\omega$

*** Problem 19-40**

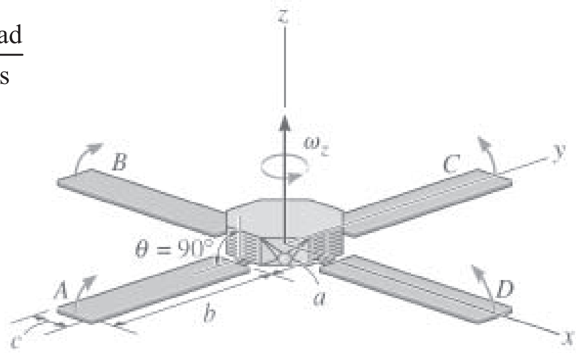
The space satellite has mass m_{ss} and moment of inertia I_z, excluding the four solar panels *A,B,C,* and *D*. Each solar panel has mass m_p and can be approximated as a thin plate.If the satellite is originally spinning about the *z* axis at aconstant rate ω_z, when $\theta = 90°$, determine the rate of spin if all the panels are raised and reach the upward position, $\theta = 0°$, at the same instsnt.

Given:

$m_{ss} = 125\ \text{kg} \qquad a = 0.2\ \text{m} \qquad \omega_z = 0.5\ \dfrac{\text{rad}}{\text{s}}$

$I_z = 0.940\ \text{kg}\cdot\text{m}^2 \qquad b = 0.75\ \text{m}$

$m_{sp} = 20\ \text{kg} \qquad c = 0.2\ \text{m}$



Solution: Guess $\omega_{z2} = 1\ \dfrac{\text{rad}}{\text{s}}$

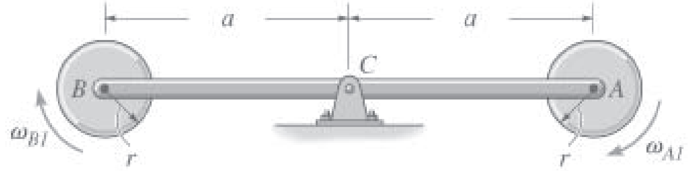
Given

$$\left[I_z + 4\left[\frac{m_{sp}}{12}\left(b^2 + c^2\right) + m_{sp}\left(a + \frac{b}{2}\right)^2\right]\right]\omega_z = \left[I_z + 4\left(\frac{m_{sp}}{12}c^2 + m_{sp}\,a^2\right)\right]\omega_{z2}$$

$\omega_{z2} = \text{Find}\left(\omega_{z2}\right) \qquad \omega_{z2} = 3.56\ \dfrac{\text{rad}}{\text{s}}$

Problem 19-41

Rod *ACB* of mass m_r supports the two disks each of mass m_d at its ends. If both disks are given a clockwise angular velocity $\omega_{A1} = \omega_{B1} = \omega_0$ while the rod is held stationary and then released, determine the angular velocity of the rod after both disks have stopped spinning relative to the rod due to frictional resistance at the pins *A* and *B*. Motion is in the *horizontal plane.* Neglect friction at pin *C*.



Given:

$m_r = 2\ \text{kg}$

$m_d = 4\ \text{kg}$

$\omega_0 = 5\ \frac{\text{rad}}{\text{s}}$

$a = 0.75\ \text{m}$

$r = 0.15\ \text{m}$

Solution:

$$2\left(\frac{1}{2}m_d\right)r^2\omega_0 = \left[2\left(\frac{1}{2}m_d\right)r^2 + 2m_d a^2 + \frac{m_r}{12}(2a)^2\right]\omega_2$$

$$\omega_2 = \frac{m_d r^2}{m_d r^2 + 2m_d a^2 + \left(\frac{m_r}{3}\right)a^2}\omega_0 \qquad \omega_2 = 0.0906\ \frac{\text{rad}}{\text{s}}$$

Problem 19-42

Disk *A* has a weight W_A. An inextensible cable is attached to the weight *W* and wrapped around the disk.The weight is dropped distance *h* before the slack is taken up. If the impact is perfectly elastic, i.e., $e = 1$, determine the angular velocity of the disk just after impact.

Given:

$W_A = 20\ \text{lb} \qquad h = 2\ \text{ft}$

$W = 10\ \text{lb} \qquad r = 0.5\ \text{ft}$

Solution:

$v_1 = \sqrt{2gh}$

Guess $\qquad \omega_2 = 1\ \frac{\text{rad}}{\text{s}} \qquad v_2 = 1\ \frac{\text{ft}}{\text{s}}$

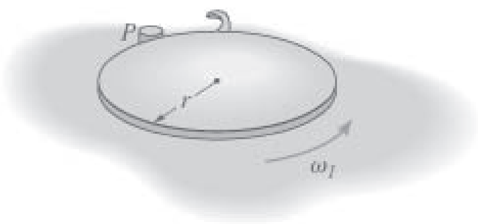
Given $\left(\frac{W}{g}\right)v_1 r = \left(\frac{W}{g}\right)v_2 r + \left(\frac{W_A}{g}\right)\frac{r^2}{2}\omega_2 \qquad v_2 = \omega_2 r$

$$\begin{pmatrix} v_2 \\ \omega_2 \end{pmatrix} = \mathrm{Find}(v_2, \omega_2) \qquad v_2 = 5.67\,\frac{\mathrm{ft}}{\mathrm{s}} \qquad \omega_2 = 11.3\,\frac{\mathrm{rad}}{\mathrm{s}}$$

Problem 19-43

A thin disk of mass m has an angular velocity ω_1 while rotating on a smooth surface. Determine its new angular velocity just after the hook at its edge strikes the peg P and the disk starts to rotate about P without rebounding.

Solution:



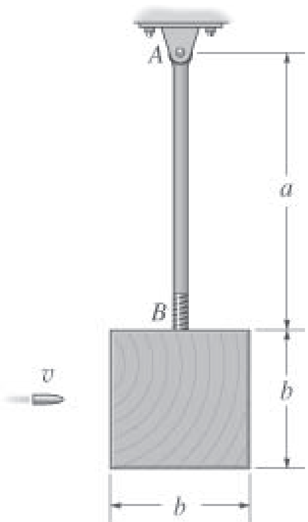
$$H_1 = H_2$$

$$\left(\frac{1}{2}m r^2\right)\omega_1 = \left(\frac{1}{2}m r^2 + m r^2\right)\omega_2$$

$$\omega_2 = \frac{1}{3}\omega_1$$

***Problem 19-44**

The pendulum consists of a slender rod AB of weight W_r and a wooden block of weight W_b. A projectile of weight W_p is fired into the center of the block with velocity v. If the pendulum is initially at rest, and the projectile embeds itself into the block, determine the angular velocity of the pendulum just after the impact.



Given:

$W_r = 5\ \mathrm{lb} \quad W_p = 0.2\ \mathrm{lb} \quad v = 1000\,\frac{\mathrm{ft}}{\mathrm{s}} \quad g = 32.2\,\frac{\mathrm{ft}}{\mathrm{s}^2}$

$W_b = 10\ \mathrm{lb} \quad a = 2\ \mathrm{ft} \quad b = 1\ \mathrm{ft}$

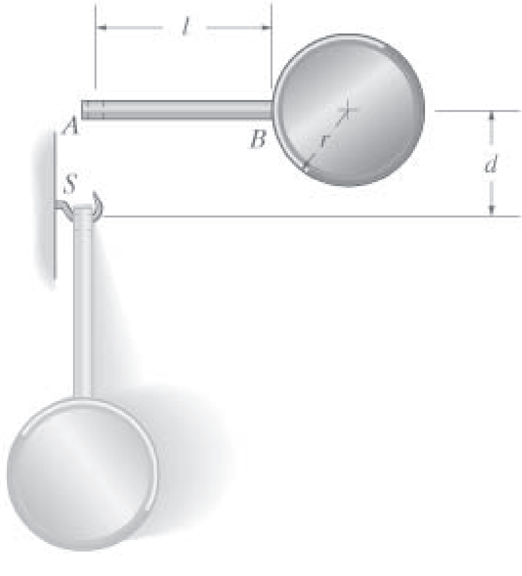
Solution:

$$\left(\frac{W_p}{g}\right)v\left(a + \frac{b}{2}\right) = \left[\left(\frac{W_r}{g}\right)\left(\frac{a^2}{3}\right) + \left(\frac{W_b}{g}\right)\left(\frac{b^2}{6}\right) + \left(\frac{W_b}{g}\right)\left(a + \frac{b}{2}\right)^2 + \left(\frac{W_p}{g}\right)\left(a + \frac{b}{2}\right)^2\right]\omega_2$$

$$\omega_2 = \frac{W_p v\left(a + \frac{b}{2}\right)}{W_r\frac{a^2}{3} + W_b\frac{b^2}{6} + W_b\left(a + \frac{b}{2}\right)^2 + W_p\left(a + \frac{b}{2}\right)^2} \qquad \omega_2 = 6.94\,\frac{\mathrm{rad}}{\mathrm{s}}$$

Problem 19-45

The pendulum consists of a slender rod AB of mass M_1 and a disk of mass M_2. It is released from rest without rotating. When it falls a distance d, the end A strikes the hook S, which provides a permanent connection. Determine the angular velocity of the pendulum after it has rotated 90°. Treat the pendulum's weight during impact as a nonimpulsive force.



Given:

$M_1 = 2\text{kg} \qquad r = 0.2\text{m}$

$M_2 = 5\text{kg} \qquad l = 0.5\text{m}$

$d = 0.3\text{m}$

Solution:

$$v_1 = \sqrt{2gd}$$

$$I_A = M_1\frac{l^2}{3} + M_2\frac{r^2}{2} + M_2(l+r)^2$$

Guesses

$$\omega_2 = 1\frac{\text{rad}}{\text{s}} \qquad \omega_3 = 1\frac{\text{rad}}{\text{s}}$$

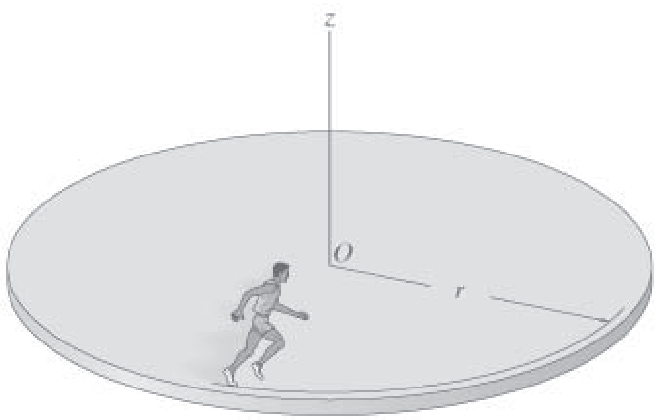
Given

$$M_1 v_1\frac{l}{2} + M_2 v_1(l+r) = I_A\omega_2 \qquad \frac{1}{2}I_A\omega_2^{\,2} = \frac{1}{2}I_A\omega_3^{\,2} - M_1 g\frac{l}{2} - M_2 g(l+r)$$

$$\begin{pmatrix}\omega_2\\ \omega_3\end{pmatrix} = \text{Find}(\omega_2, \omega_3) \qquad \omega_2 = 3.57\frac{\text{rad}}{\text{s}} \qquad \omega_3 = 6.46\frac{\text{rad}}{\text{s}}$$

Problem 19-46

A horizontal circular platform has a weight W_1 and a radius of gyration k_z about the z axis passing through its center O. The platform is free to rotate about the z axis and is initially at rest. A man having a weight W_2 begins to run along the edge in a circular path of radius r. If he has a speed v and maintains this speed relative to the platform, determine the angular velocity of the platform. Neglect friction.



Given:

$W_1 = 300 \text{ lb} \quad v = 4 \frac{\text{ft}}{\text{s}}$

$W_2 = 150 \text{ lb}$

$r = 10 \text{ ft} \qquad k_z = 8 \text{ ft}$

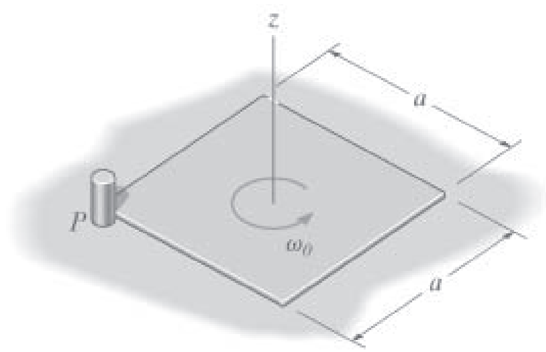
Solution:

$M v r = I\omega$

$\frac{W_2}{g} v r = \frac{W_1}{g} k_z^2 \omega \qquad \omega = W_2 v \frac{r}{W_1 k_z^2} \qquad \omega = 0.312 \frac{\text{rad}}{\text{s}}$

Problem 19-47

The square plate has a weight W and is rotating on the smooth surface with a constant angular velocity ω_0. Determine the new angular velocity of the plate just after its corner strikes the peg P and the plate starts to rotate about P without rebounding.

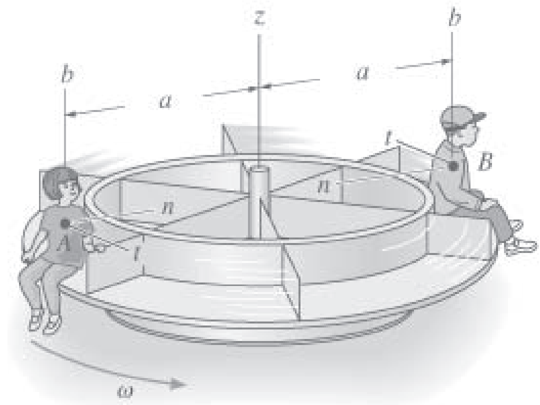


Solution:

$\left(\frac{W}{g}\right)\left(\frac{a^2}{6}\right)\omega_0 = \left(\frac{W}{g}\right)\left(\frac{2a^2}{3}\right)\omega \qquad \omega = \frac{1}{4}\omega_0$

***Problem 19-48**

Two children A and B, each having a mass M_1, sit at the edge of the merry-go-round which is rotating with angular velocity ω. Excluding the children, the merry-go-round has a mass M_2 and a radius of gyration k_z. Determine the angular velocity of the merry-go-round if A jumps off horizontally in the $-n$ direction with a speed v, measured with respect to the merry-go-round. What is the merry-go-round's angular velocity if B then jumps off horizontally in the $+t$ direction with a speed v, measured with respect to the merry-go-round? Neglect friction and the size of each child.



Given:

$M_1 = 30\ \text{kg}$ $\quad k_z = 0.6\ \text{m}$

$M_2 = 180\ \text{kg}$ $\quad a = 0.75\ \text{m}$

$\omega = 2\ \frac{\text{rad}}{\text{s}}$ $\quad v = 2\ \frac{\text{m}}{\text{s}}$

Solution:

(a) Guess $\omega_2 = 1\ \frac{\text{rad}}{\text{s}}$

Given $\left(M_2 k_z^2 + 2M_1 a^2\right)\omega = \left(M_2 k_z^2 + M_1 a^2\right)\omega_2$

$\omega_2 = \text{Find}(\omega_2)$ $\quad \omega_2 = 2.41\ \frac{\text{rad}}{\text{s}}$

(b) Guess $\omega_3 = 1\ \frac{\text{rad}}{\text{s}}$

Given $\left(M_2 k_z^2 + M_1 a^2\right)\omega_2 = M_2 k_z^2 \omega_3 + M_1\left(v + \omega_3 a\right)a$

$\omega_3 = \text{Find}(\omega_3)$ $\quad \omega_3 = 1.86\ \frac{\text{rad}}{\text{s}}$

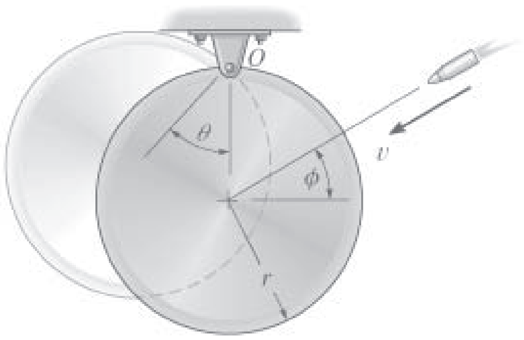
Problem 19-49

A bullet of mass m_b having velocity v is fired into the edge of the disk of mass m_d as shown. Determine the angular velocity of the disk of mass m_d just after the bullet becomes embedded in it. Also, calculate how far θ the disk will swing until it stops. The disk is originally at rest.

Given:

$m_b = 7\ \text{gm}$ $\quad m_d = 5\ \text{kg}$ $\quad \phi = 30\ \text{deg}$

$v = 800\ \frac{\text{m}}{\text{s}}$ $\quad r = 0.2\ \text{m}$



Solution:

Guesses $\omega = 1\ \frac{\text{rad}}{\text{s}}$ $\quad \theta = 10\ \text{deg}$

Given $m_b v \cos(\phi) r = \frac{3}{2} m_d r^2 \omega$ $\qquad -m_d g r + \frac{1}{2}\left(\frac{3}{2} m_d r^2\right)\omega^2 = -m_d g r \cos(\theta)$

$\begin{pmatrix}\omega \\ \theta\end{pmatrix} = \text{Find}(\omega, \theta)$ $\quad \omega = 3.23\ \frac{\text{rad}}{\text{s}}$ $\quad \theta = 32.8\ \text{deg}$

Problem 19-50

The two disks each have weight W. If they are released from rest when $\theta = \theta_1$, determine the maximum angle θ_2 after they collide and rebound from each other. The coefficient of restitution is e. When $\theta = 0°$ the disks hang so that they just touch one another.

Given:

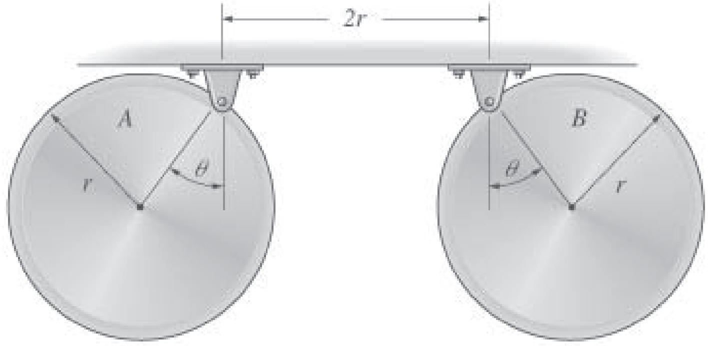
$$g = 32.2 \frac{\text{ft}}{\text{s}^2}$$

$$W = 10 \text{ lb}$$

$$\theta_1 = 30 \text{ deg}$$

$$e = 0.75$$

$$r = 1 \text{ ft}$$



Solution:

Guesses $\omega_1 = 1 \frac{\text{rad}}{\text{s}}$ $\omega_2 = 1 \frac{\text{rad}}{\text{s}}$ $\theta_2 = 10\text{deg}$

Given $$-W r\cos(\theta_1) = \frac{1}{2}\left(\frac{3}{2}\frac{W}{g}r^2\right)\omega_1^{\,2} - W r$$

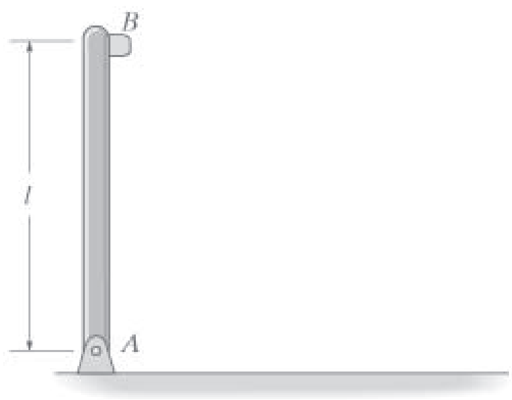
$$e r\omega_1 = r\omega_2$$

$$-W r + \frac{1}{2}\left(\frac{3}{2}\frac{W}{g}r^2\right)\omega_2^{\,2} = -W r\cos(\theta_2)$$

$$\begin{pmatrix}\omega_1\\ \omega_2\\ \theta_2\end{pmatrix} = \text{Find}(\omega_1, \omega_2, \theta_2) \qquad \begin{pmatrix}\omega_1\\ \omega_2\end{pmatrix} = \begin{pmatrix}2.40\\ 1.80\end{pmatrix}\frac{\text{rad}}{\text{s}} \qquad \theta_2 = 22.4\text{ deg}$$

Problem 19-51

The rod AB of weight W is released from rest in the vertical position. If the coefficient of restitution between the floor and the cushion at B is e, determine how high the end of the rod rebounds after impact with the floor.



Given:

$$W = 15 \text{ lb}$$

$$l = 2 \text{ ft}$$

$$e = 0.7$$

Solution:

Guesses $\omega_1 = 1\frac{\text{rad}}{\text{s}}$ $\omega_2 = 1\frac{\text{rad}}{\text{s}}$ $\theta = 1\text{deg}$

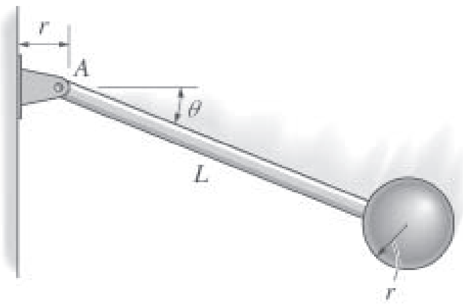
Given

$$W\left(\frac{l}{2}\right) = \frac{1}{2}\left(\frac{W}{g}\right)\frac{l^2}{3}\omega_1^{\,2} \qquad e\omega_1 l = \omega_2 l \qquad W\left(\frac{l}{2}\right)\sin(\theta) = \frac{1}{2}\left(\frac{W}{g}\right)\frac{l^2}{3}\omega_2^{\,2}$$

$$\begin{pmatrix}\omega_1\\ \omega_2\\ \theta\end{pmatrix} = \text{Find}(\omega_1, \omega_2, \theta) \qquad \begin{pmatrix}\omega_1\\ \omega_2\end{pmatrix} = \begin{pmatrix}6.95\\ 4.86\end{pmatrix}\frac{\text{rad}}{\text{s}} \qquad \theta = 29.34\text{ deg}$$

$$h = l\sin(\theta) \qquad h = 0.980\text{ ft}$$

***Problem 19-52**



The pendulum consists of a solid ball of weight W_b and a rod of weight W_r. If it is released from rest when $\theta_1 = 0°$, determine the angle θ_2 after the ball strikes the wall, rebounds, and the pendulum swings up to the point of momentary rest.

Given:

$W_b = 10\text{lb}$ $e = 0.6$ $L = 2\text{ft}$

$W_r = 4\text{lb}$ $r = 0.3\text{ft}$ $g = 32.2\frac{\text{ft}}{\text{s}^2}$

Solution:

$$I_A = \left(\frac{W_r}{g}\right)\left(\frac{L^2}{3}\right) + \frac{2}{5}\left(\frac{W_b}{g}\right)r^2 + \frac{W_b}{g}(L+r)^2$$

Guesses $\omega_1 = 1\frac{\text{rad}}{\text{s}}$ $\omega_2 = 1\frac{\text{rad}}{\text{s}}$ $\theta_2 = 10\text{deg}$

Given $$0 = -W_b(L+r) - W_r\left(\frac{L}{2}\right) + \frac{1}{2}I_A\omega_1^{\,2}$$

$$e(L+r)\omega_1 = (L+r)\omega_2$$

$$-W_b(L+r) - W_r\left(\frac{L}{2}\right) + \frac{1}{2}I_A\omega_2^{\,2} = -\left[W_b(L+r) + W_r\left(\frac{L}{2}\right)\right]\sin(\theta_2)$$

$$\begin{pmatrix}\omega_1\\ \omega_2\\ \theta_2\end{pmatrix} = \text{Find}(\omega_1, \omega_2, \theta_2) \qquad \begin{pmatrix}\omega_1\\ \omega_2\end{pmatrix} = \begin{pmatrix}5.45\\ 3.27\end{pmatrix}\frac{\text{rad}}{\text{s}} \qquad \theta_2 = 39.8\text{ deg}$$

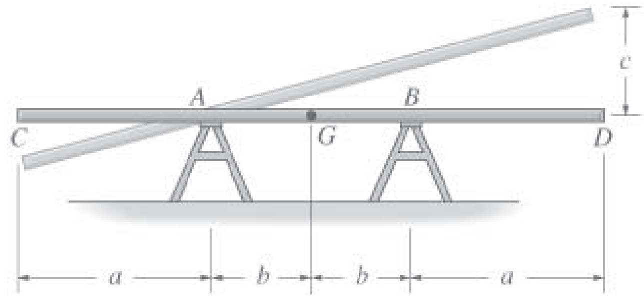
Problem 19-53

The plank has a weight W, center of gravity at G, and it rests on the two sawhorses at A and B. If the end D is raised a distance c above the top of the sawhorses and is released from rest, determine how high end C will rise from the top of the sawhorses after the plank falls so that it rotates clockwise about A, strikes and pivots on the sawhorses at B, and rotates clockwise off the sawhorse at A.

Given:

$W = 30\text{lb}$ $b = 1.5\text{ft}$

$a = 3\text{ft}$ $c = 2\text{ft}$



Solution:

$$I_G = \frac{1}{12}\left(\frac{W}{g}\right)4(a+b)^2 \qquad I_A = I_G + \left(\frac{W}{g}\right)b^2$$

Guesses $\omega_1 = 1\frac{\text{rad}}{\text{s}}$ $\omega_2 = 1\frac{\text{rad}}{\text{s}}$ $\theta = 1\text{deg}$ $h = 1\text{ft}$

Given $$W\left(\frac{b}{2b+a}\right)c = \frac{1}{2}I_A\omega_1^{\,2} \qquad -I_G\,\omega_1 + \left(\frac{W}{g}\right)\omega_1\,b\,b = -I_A\,\omega_2$$

$$W\,b\sin(\theta) = \frac{1}{2}I_A\omega_2^{\,2} \qquad h = (a+2b)\sin(\theta)$$

$$\begin{pmatrix}\omega_1\\ \omega_2\\ \theta\\ h\end{pmatrix} = \text{Find}(\omega_1, \omega_2, \theta, h) \qquad \begin{pmatrix}\omega_1\\ \omega_2\end{pmatrix} = \begin{pmatrix}1.89\\ 0.95\end{pmatrix}\frac{\text{rad}}{\text{s}} \qquad \theta = 4.78\,\text{deg} \qquad h = 0.500\,\text{ft}$$

Problem 19-54

Tests of impact on the fixed crash dummy are conducted using the ram of weight W that is released from rest at $\theta = \theta_1$ and allowed to fall and strike the dummy at $\theta = \theta_2$. If the coefficient of restitution between the dummy and the ram is e, determine the angle θ_3 to which the ram will rebound before momentarily coming to rest.

Given:

$W = 300\text{ lb}$ $e = 0.4$

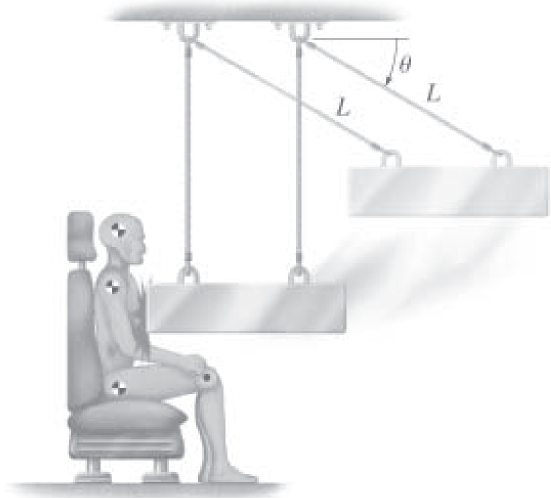
$\theta_1 = 30\text{ deg}$ $L = 10\text{ ft}$

$\theta_2 = 90\text{ deg}$ $g = 32.2\,\frac{\text{ft}}{\text{s}^2}$

Solution:

Guesses $v_1 = 1\frac{\text{ft}}{\text{s}}$ $v_2 = 1\frac{\text{ft}}{\text{s}}$

$\theta_3 = 1\ \text{deg}$



Given

$$-W L \sin(\theta_1) = \frac{1}{2}\left(\frac{W}{g}\right) v_1^2 - W L \sin(\theta_2)$$

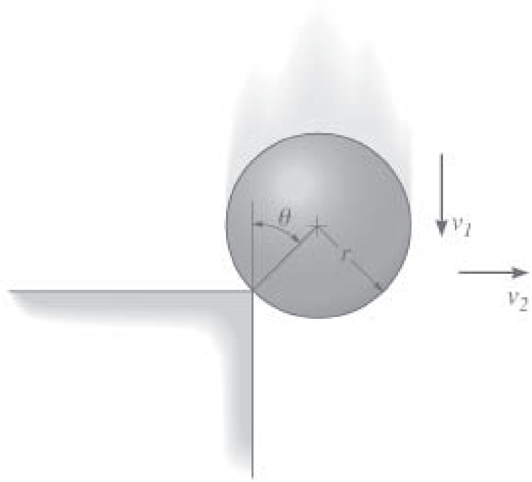
$$-W L \sin(\theta_2) + \frac{1}{2}\left(\frac{W}{g}\right) v_2^2 = -W L \sin(\theta_3)$$

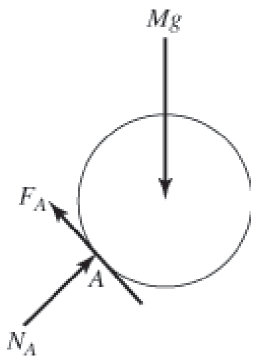
$$e v_1 = v_2$$

$$\begin{pmatrix} v_1 \\ v_2 \\ \theta_3 \end{pmatrix} = \text{Find}(v_1, v_2, \theta_3) \qquad \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 17.94 \\ 7.18 \end{pmatrix}\frac{\text{ft}}{\text{s}} \qquad \theta_3 = 66.9\ \text{deg}$$

Problem 19-55

The solid ball of mass m is dropped with a velocity v_1 onto the edge of the rough step. If it rebounds horizontally off the step with a velocity v_2, determine the angle θ at which contact occurs. Assume no slipping when the ball strikes the step. The coefficient of restitution is e.





Solution:

No slip $\omega_2 r = v_2 \cos(\theta)$

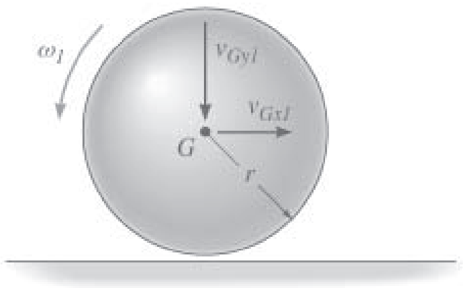
Angular Momentum about *A* $m v_1 r \sin(\theta) = m v_2 r \cos(\theta) + \frac{2}{5} m r^2 \omega_2$

Restitution $e v_1 \cos(\theta) = v_2 \sin(\theta)$

Combining we find $\theta = \operatorname{atan}\left(\sqrt{\frac{7e}{5}}\right)$

***Problem 19-56**

A solid ball with a mass *m* is thrown on the ground such that at the instant of contact it has an angular velocity ω_1 and velocity components v_{Gx1} and v_{Gy1} as shown. If the ground is rough so no slipping occurs, determine the components of the velocity of its mass center just after impact. The coefficient of restitution is *e*.



Solution:

Restitution $e v_{Gy1} = v_{Gy2}$

Angular Momentum $\frac{2}{5} m r^2 \omega_1{}^2 - m v_{Gx1} r = \frac{2}{5} m r^2 \omega_2 + m v_{Gx2} r$

No slip $v_{Gx2} = \omega_2 r$

Combining $v_{G2} = \begin{pmatrix} \frac{5}{7} v_{Gx1} - \frac{2}{7} r \omega_1 \\ e v_{Gy1} \end{pmatrix}$
